

Notes on

Vagueness

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Content

0. Preliminary Remarks
1. The Source of Vagueness
2. Language Practice and 3-valuedness
3. The Logic of Vagueness
4. Sorites, Borderline Cases, and other Issues
5. Criticism and Comparison of Approaches
6. References

§0 Preliminary Remarks

0.1 These notes neither provide an overview nor an introduction to theories of vagueness. (There is excellent literature which does that – see the Reference section at the end.)

0.2 These notes engage with views and arguments found in the literature on vagueness, and put forth a view on vagueness.

0.3 This view sees vagueness as a central feature of natural languages. And it – ultimately – has to be combined with another feature of semantically closed natural languages: contradictions. The result will be a 4-valued logic which brings its own intricacies and meta-logical issues. The considerations here deal only with vagueness. The debate about reference failure and Free Logic should also be kept apart from the debate about vagueness.

0.4 Despite the criticism brought forward against 3-valued approaches to vagueness the view seems to me the best approach to vagueness, given a foundation in philosophical semantics. The approach here involves both a Boolean negation and a predicate negation. One could also understand the approach as a 3-valued version of an epistemic account of vagueness in its recognition of boundaries. These boundaries are based on linguistic practice, so the 3rd value is semantic (not lack of knowledge).

0.5 Often with philosophical analyses no single analysis and model satisfies completely all demands one may have on a perfect analysis. In this situation one has to weigh the strains each analysis puts on finding the (best) intended solution against each other. I argue that although some version of a 3-valued analysis puts some strain on our acceptance of its analysis of vagueness it does so to a lesser extent than rival analyses. Unfortunately, participants in the debate often look at the other's theses with the proverbial 'incredulous stare'.

§1 The Source of Vagueness

1.1 *Semantic indecision* Is a ubiquitous phenomenon of natural languages and thus has to be admitted into an analysis and model of natural language logic.

1.1.1 In ordinary discourse we can find ourselves in a position that we can't say how to evaluate a vague predicate [P] with respect to some item a .¹ Given a vague predicate it can be indeterminate whether it applies. Any such *semantic* indecision has to be distinguished from epistemic ignorance as to a proper evaluation. This may even occur in case of non-vague expression in case we have incomplete access to confirming or refuting evidence. Epistemic indecision can be expressed (using a BELIEF and a TRUE operator) as:

$$\neg \text{BELIEF}(A) \wedge \neg \text{BELIEF}(\neg A) \wedge \text{BELIEF}(\text{TRUE}(A) \vee \text{TRUE}(\neg A)).$$

Semantic indecision involves that (some crucial form of) *tertium non datur* (TND) does not apply to vague expressions.

1.2 Vagueness is ubiquitous in natural languages, because of our limited discriminatory abilities. Words are employed on occasions of usage which are similar to a degree. Users accommodate to a linguistic practice by trying to follow those practices given their own standards of sufficient similarity between the occasions of usage. Terms, thus, have a range of usage, on the fringe of that range are the borderline cases.

1.2.1 This applies to the positive range of a term as well as to the range of the predicate negation of the term.

1.2.2 Associated with a term are criteria for its employment, although they are not its meaning and may be manifold and not all socially shared. Fulfilment of the criteria establishes a positive case, although this fulfilment may itself not be clear cut, thus generating a borderline case. This also holds for criteria of employing the predicate negation, which may be

¹ These notes focus on vague predicates and adjectives only. Square brackets are used as corner quotes. The symbolism should be familiar. In the logical system we look only at the propositional level. Sub-sentential structure in elementary sentences is mentioned only in the development of the theory. Predicate [P] corresponds to property P. Singular term [a] to item a. Predicates in the technical sense include open sentences and adjectives.

independent of the positive criteria. If neither the positive nor the negative criteria apply we have an indeterminate case.

1.3 Indeterminate cases are a semantic phenomenon, not an epistemic. An indeterminate case stays indeterminate (unless usage changes). Vagueness is ineliminable.

1.3.1 Category mistakes are a semantic phenomenon, as well, but are not cases of vagueness or indeterminacy: in case of a category mistake the criteria of predicate application *entail* that the predicate cannot apply to some type of object, so that it is *a semantic truth* ('analytic') that these applications are *false*. "7 is green" is not determinate, but false, because "It is not the case that numbers are coloured" is analytic.

1.3.1.1 Similar reflections show that presupposition failure – by most linguists considered to be a pragmatic phenomenon anyway – must be distinguished from vagueness.

1.4 With respect to predicates one can distinguish the positive case $[P(a)]$ and the explicit predicate negation ('strong negation') $\sim P(a)$, and the intermediate range of indeterminate cases.

1.4.1 Predicates have an extension. Non-vague predicates divide the domain into their extension and the rest (of all objects for which the application of the predicate is *false*). Vague predicates have an extension, an anti-extension (of all objects for which the *predicate negation* applied to the predicate is *true*), and a range in between (of all objects which are indeterminate with respect to the application of the predicate). In formal semantics one will model this generally by an interpretation of a predicate assigning a positive extension, a negative extension, and a range of indeterminacy, which is empty for non-vague predicates.

1.5 As terms apply to a *range* of objects *those* may be constituted in different ways that disjointly are covered by the term. Some constitutions resemble one another in their effects so that they fall under the same term. Some resemble objects that fall under the term, but not close enough that the term applies. Such objects are borderline cases of term employment. These objects are not vague. They just are what they are. Terms are vague. There are no vague objects. Vagueness resides in language.

1.5.1 That a predicate $[P]$ is vague does not mean that there is a vague property P . To talk this way is – at least – misleading: If a predicate is vague, it covers a range of object constitution disjunctively, one may better call the property P 'disjunctively constituted by language use'. Of course, the constitution of each object falling under $[P]$ is not constituted by language. Language use collects differently constituted object under a common predicate.

1.6 Whereas borderline cases may neither fulfil the positive nor the negative criteria of a term's employment on occasion some item may fulfil both resulting in an inconsistent object described by a contradiction. Apart from cases in logic and semantics, where dialetheism seems

unavoidable, such cases are typically an occasion to sharpen the criteria, so that contradictory objects can be avoided. Positive and negative extensions assigned by an interpretation to a predicate should not overlap.

§2 Language Practice and 3-valuedness

2.1 For any descriptive vague term of a language there is a range for which, say, 90 % of speakers apply the term, and a range for which, say, 90 % of speakers apply the predicate negation of the term. This *could* be established by a survey, in principle. Even if we *do not* conduct the survey, there will be (there is) this range of application, as well as the range of applying the predicate negation. So, we can in semantic analysis work with the theoretical entity ‘cut off point’ though we do not actually know where to place it. Accordingly, we are justified in assuming the *existence* of such statistical cut off point for application and predicate negation. Cut offs exist (in linguistic practice), even if we do not recognize them.

2.1.1 We are thus justified to model usage on a 3-valued basis. The indeterminate cases being those falling between the cut off points. Language usage does not establish a verdict on them, although a small group of speakers may have opinions on them.

2.1.1.1 The positive cases form a set, the negative cases form a set, the indeterminate cases form a set. Sets are not vague. We may be semantically unsure in which set to place an item, but given the cut offs the item either belongs to the predicate’s extension or it does not. Set membership is not vague.

2.1.1.2 The 3-valued logic covered here is fully compatible with ZFC, and has no track with ‘fuzzy sets’.

2.1.2 Part of the agreed opinion on vagueness in a natural language linguistic community is (i) the existence of indeterminate cases, (ii) the failure of TND for predicate negation, (iii) the availability of a Boolean negation, for which TND holds.

2.1.2.1 On non-vague sentences Boolean and predicate negation coincide. On vague sentences their logic differs.

2.1.3 Predicate negation can be present in a language by (i) negation particles (like “in-” or “un-”), and (ii) contrast pairs (like “tall” and “small”). Boolean negation is typically expressed by “it is not the case that”.

2.1.4 Allowing for a range of indeterminate cases and recognizing the existence of predicate negation in natural language does more justice to usage than to draw a single $F/\neg F$ boundary only. We can introduce a Boolean negation – if not taken as present in language, anyway.

Speakers, however, agree on the existence of vague expressions as well, and thus assume a range of indetermination and the difference between predicate and Boolean negation. We can use a Boolean negation – especially in formal languages – and deal with inferences in standard logic. If we want to capture natural language negation, however, the picture is more involved.

2.2 If cut off points are established as statistical regularities of usage, they can be surprising for individual speakers. Cut off points are social. They reflect the usage in a language community. Patterns of usage exist partially ‘behind the backs’ of individual users.

2.2.1 The boundaries between P , I , $\sim P$ are not observable. They are built into patterns of usage. We can imagine a long-term comprehensive study which identifies the set P of items 90% of users classify as positive cases, correspondingly with $\sim P$, all other items being I with respect to P . That we do not actually discover the boundaries does not mean that they are not there. Their mere existence suffices to model the logic of vagueness. From an individual speaker’s perspective some items are indeterminate and this has to be taken into account when reasoning about them. An individual may even deviate from the statistical regularity defining the boundaries. This may happen, of course, as the boundaries are not observable in an individual situation of use. This is not more surprising or devastating for the existence of such boundaries than similar misapprehensions in any other case of social statistics. Someone may consider herself above average income – even with some reason – when a statistical data survey no one has actually conducted would reveal that she is not. Statistical regularities show themselves in patterns of language usage. Attuning to such patterns constitutes integration into the linguistic community.

2.2.1.1 Fixing a threshold of agreeing speakers fixes corresponding boundaries. The exact threshold is a conventional choice. This, however, does not mean that the function from thresholds to boundaries is arbitrary. This function is determined by actual usage. What is conventional, thus, is only the *selection* of some specific threshold condition as paradigmatic or definitional. The related boundary was there before. In principle it could be discovered. That fixing the meaning of an expression involves some convention does not distinguish vague predicates from any others. The convention and the idea of a threshold determining von observed boundaries may be more involved than in cases of explicit definitions, say in case of artefacts. This should not be misunderstood as rendering some meaning ‘unnatural’. In that sense meanings of many non-vague expressions are ‘unnatural’, if not most meanings.

2.2.2 If there is some cut off point for [tadpole] thus established by a statistical threshold, there will be an animal such that for some very short interval the animal is a tadpole according to the semantic rules of that language and is not a tadpole very shortly after. This seems counterintuitive or paradoxical as the change or time interval is so small, but it is not paradoxical, but expresses the practice (statistical regularities) of a language community.

2.3 Such a statistical procedure resembles the idea of supervenience in that it establishes a common ground *over* individual uses (of different individuals or individuals over time). It is highly unlikely that 90% of speakers themselves agree on all cases (as with disagreement in ‘precifications’ of supervenience). Established usage is in its details a hidden statistical regularity, *reflected* in common/daily agreements and working linguistic practice.

2.3.1 Established usage serves as a norm in a communication context. On occasion even a large majority of users can be wrong about a term’s application. (This serves as a hallmark of the normativity of meaning in philosophical semantics.) Over *repeated* occasions where almost all speakers (say, our 90%) deviate from established usage the term’s usage shifts and the new usage becomes established usage, but it is wrong to assume that agreement of enough competent speakers on an occasion (e.g. one Sorites interview session) determines an indeterminate case.

2.3.2 By defining a threshold (say, 90 % of speakers) for the clear cases A and ~A all other cases can be treated as indeterminate without implying that there are single borders for all users.

2.3.3 The whole discussion of the absence of boundaries seems too close to imagery from observable (e.g. customs) boundaries between nation states. Boundaries need not be known by us to be there.

§3 The Logic of Vagueness

3.1 The logic of vagueness is a 3-valued logic. Let us call it “PCV” (for “Classical Propositional Calculus + Vagueness”). The 3 truth values are: True (T), False (F), Indeterminate (I). For PCV *Quartus Non Datur*.

3.1.1 One can introduce unary connectives for the truth values, which are mutually exclusive, by their obvious truth tables. This way one can express the status of indeterminacy *directly* in the language. (As long as we – on the propositional level – do not talk about truth values as items themselves, the truth value connectives can be homophonous to the truth values.)

A	TA	FA	IA
T	T	F	F
F	F	T	F
I	F	F	T

3.1.2 The language of PCV extends the language of PC by a second negation $[\sim]$, with the obvious extension of formation rules. That is the crucial addition, the truth value operators (above) and (below) a new equivalence connective $[\equiv]$ are optional.

3.1.2.1 $[\neg]$ turns an indeterminate sentence in a true sentence (i.e. the Boolean negation of an I sentence is T). $[\neg A]$ means A being not-true (i.e. covers both I and F).

3.1.3 The logical impact of vagueness can be seen at the propositional level.

3.2 The semantics of vagueness can be laid down by 3-valued truth tables.

3.2.1 The basic semantic (and epistemic) value is truth. In inference we want to preserve truth, and a conditional cannot be true if we get from a true antecedent to a false consequent. Thus, in PC this is the only case where $A \supset B$ is false.² Indeterminateness is semantically of less value than truth. We will not, for instance, assert a sentence we know just to be I. We cannot detach from an I antecedent. ‘Moving’ from T to I should be I itself, not T. We will actually never ‘move’ this way as detachment needs a true conditional. One reason to have a conditional with true antecedent and false consequent being false is to block the ‘move’. Following the promiscuous attitude of PC $[\supset]$, moving from I to F is not as bad as moving from T to F, thus should not be F, although – maybe – not be true. Again, there is no danger of detachment. Given that we never detach from F or I antecedents, one may even argue – in the promiscuous spirit – for a conditional with I antecedent and F consequent to be true. To keep with the image of I being of intermediate value between T and F one might take that conditional as I. Now, as this image is just an image, but detachment concerns the core of logic PCV follows the promiscuous strategy and considers all conditionals with an I antecedent as true. As we lose nothing by moving from I to I, this should be true, not declaring something to be not true if there is no need to do so, although – as said before – we will never ‘move’ this way. Compare PC: although conditionals with F antecedents are T, we never ‘move’ from a F antecedent anywhere by detachment. Explained from a different angle. Moving from I to F should not be F, as we do not have the crucial forbidden case here: we do not have a T antecedent. Correspondingly: moving from T to I should not be F, as we do not have the forbidden case here: we do not have a F consequent. And in the spirit of passing on semantic value as the hallmark of

² The debate to what extent $[\supset]$ can model the indicative conditional in natural language is extensive and involves not just the theory of inferences, but the theories of natural language use and communication principles. The debate cannot be conducted here, and is independent of the issue of vagueness. Although $[\supset]$ does not cover many intricacies of natural language indicative conditionals, and some of the ‘paradoxes’ of $[\supset]$ should be avoided at some point (e.g. in a semantics without ECQL), the case for $[\supset]$ being sufficiently close to indicative conditionals in natural language is strong, and so we proceed here like in PC and the standard theory of deduction on that assumption. A full analysis of natural language logic may need a further conditional connective.

conditional reasoning, moving from I to I should be accepted, even if it is excluded in practice, as only truth gets us ‘moving’.

3.2.1.1 These considerations motivate best the following truth table for a 3-valued $\supset$ – best among other alternatives, like I afflicting completely any sentence which has an I sub-sentence.

$\supset$	T	F	I
T	T	F	I
F	T	T	T
I	T	T	T

3.2.2 Truth tables for $\neg$ and $\sim$ are obvious. Disjunction and conjunction tables follow the ideas above that being I is ‘better’ than being F, but not as ‘good’ as being T. (There has been far less debate on these tables than on the table for $\supset$.)

$\wedge$	T	F	I
T	T	F	I
F	F	F	F
I	I	F	I

$\vee$	T	F	I
T	T	T	T
F	T	F	I
I	T	I	I

A	$\neg A$	$\sim A$
T	F	F
F	T	T
I	T	I

3.2.3 Given the definition of $A \equiv B$ as $(A \supset B) \wedge (B \supset A)$ the sentence $A \equiv B$ will be true in case one of the sentences is false and the other indeterminate. One could introduce a new connective $[\equiv]$ which is true only in case both sides have the same truth value.

$\equiv$	T	F	I
T	T	F	F
F	F	T	F
I	F	F	T

We leave basic PCV without $[\equiv]$

3.3 The logic of vagueness can be captured by a tree calculus or tableaux. The tableau rules can be read off the truth tables. The symbolism (including the branching) should be familiar. These tableaux constitute a calculus for PCV, which is sound and complete with respect to PCV semantics as laid down in the truth tables. Validity of inferences and theorems is decidable by developing a tree/tableau.

3.3.1 An interpretation v assigns to each atomic sentence of the language of PCV one of the 3 truth values. Complex sentences have the recursive truth conditions laid down in the truth tables.

3.3.1.1 Truth is the only designated truth value. $\Gamma \vdash A$ if all $B \in \Gamma$ being T excludes A being F or I.

3.3.2 An inference or sentence is valid if the tableau indirectly aiming at showing its non-validity closes. A tableau closes if all its branches close. A branch closes if a formula on it receives contradictory evaluations (i.e. it receives more than one of the truth values T, F, I, which are mutually exclusive). A branch also closes if a $[\neg]$ -formula is evaluated I, as this is impossible. (If we added the truth value connectives and $[\equiv]$ they could not be evaluated I either. In basic PCV we leave them to the side, not to clutter tableaux, and the exposition here.)

3.3.2.1 A refuting interpretation of a putative inference or theorem can be read out straight of an open branch of a corresponding tableau.

3.3.3 A tableau starts with evaluating the premises, if any, as T and the conclusion or sole formula as being F or being I.

3.3.4 The tableaux rules for the connectives are:

$\neg A, F$	$\neg A, T$		$\sim A, F$	$\sim A, T$	$\sim A, I$
	/	\			
A, T	A, F	A, I	A, T	A, F	A, I

$A \wedge B, T$	$A \wedge B, F$		$A \wedge B, I$		
A, T B, T	/	\	/		\
	A, F	B, F	A, T	A, I	A, I
			B, I	B, T	B, I

$A \vee B, T$		$A \vee B, F$	$A \vee B, I$		
/	\	A, F B, F	/		\
A, T	B, T		A, I	A, F	A, I
			B, F	B, I	B, I

$A \supset B, T$			$A \supset B, F$	$A \supset B, I$
/		\	A, T B, F	A, T B, I
A, F	A, I	B, T		

3.4 Examples:

Example: $A \supset B, B \supset C \vdash A \supset C$

$A \supset B, T$									
$B \supset C, T$									
$A \supset C, F$					$A \supset C, I$				
A, T C, F					A, T C, I				
A, F	A, I	B, T			A, F	A, I	B, T		
X	X	B, F	B, I	C, T	X	X	B, F	B, I	C, T
		X	X	X			X	X	X

All branches close. Transitivity as inference rule holds.

Example: $A, A \supset B \vdash B$ (*Modus Ponens*)

$A \supset B, T$						
A, T						
B, F			B, I			
A, F	A, I	B, T	A, F	A, I	B, T	
X	X	X	X	X	X	

Modus Ponens holds. (It does not in some degree logics of vagueness, like Fuzzy logic.)

Example: $A \wedge \neg A \vdash B$

Ex Contradictione Quodlibet (ECQL) a.k.a. ‘Explosion’

$A \wedge \neg A, T$									
B, F					B, I				
A, T					A, T				
$\neg A, T$					$\neg A, T$				
A, F		A, I			A, F		A, I		
X		X			X		X		

ECQL holds. PCV is not paraconsistent. It also holds in theorem form: $\vdash (A \wedge \neg A) \supset B$

Example $\vdash A \vee \neg A$

$A \vee \neg A, F$	$A \vee \neg A, I$		
A, F	A, I	A, F	A, I
$\neg A, F$	$\neg A, I$	$\neg A, I$	$\neg A, F$
A, T	X	X	A, T
X			X

All branches close. TND does hold for $[\neg]$. The second and third branch close because $[\neg A]$ cannot be indeterminate.

Example $\nvdash A \vee \sim A$

$A \vee \sim A, F$	$A \vee \sim A, I$		
A, F	A, I	A, F	A, I
$\sim A, F$	$\sim A, I$	$\sim A, I$	$\sim A, F$
A, T	A, I	A, I	A, T
X		X	X

The second branch is open. TND does not hold for $[\sim]$. So, TND holds for $[\neg]$ but not for $[\sim]$, as it should be.

3.5 In the presence of $[\neg]$ I and F both can be taken as $\neg T$. Taking $[F \text{ or } I]$ as $[\neg T]$ one regains the classical PC tableaux. So, consider a standardly valid inference $B_1 \dots B_n \vdash A$; this is valid in PC if and only if $B_1 \dots B_n \supset A$ is valid; to find a new counterexample the conditional had to be

indeterminate, as the falsity case has been excluded by having shown classical validity. The only case of a conditional to be I here is the $B_1 \dots B_n$ being T and A being I (the cases where $B_1 \dots B_n$ are not T are irrelevant in case of attempting to refute a classically valid inference). I is ‘weaker’ than F. Consequence A being F is excluded by $B_1 \dots B_n \vdash A$ being PC-valid. An evaluation of A as I will not be achievable in this case. Being [F or I] amounts to being $[\neg T]$. That A can be $[\neg T]$ when $B_1 \dots B_n$ are T is shown to fail by classical PC refutation reasoning (e.g. using tableaux with annotations [T] and $[\neg T]$). So, we have:

(Meta-Theorem) In the common language of PC and PCV:

$$\Gamma \vdash_{PC} A \text{ if and only if } \Gamma \vdash_{PCV} A$$

Classical inferences and theorems remain valid in PCV. PCV is a conservative extension of PC. This is as it should be, since natural language has ubiquitous vagueness but we customarily, nevertheless, use classical inferences (with the typical conversational constraints e.g. on relevance and avoiding ‘explosion’ by ECQL).

3.6 The next question after the (Meta-Theorem) then, obviously, is: What impact does $[\sim]$ have? And what of intermingling formula like $\neg \sim A$?

3.6.1 Many of the $[\sim]$ counterparts of $[\neg]$ validities are not valid as I sentences are involved.

Example: $\nvdash \neg \sim A \supset A$

$\neg \sim A \supset A, F$		$\neg \sim A \supset A, I$ $\neg \sim A, T$ A, I	
$\neg \sim A, T$ A, F		$\sim A, F$ A, T X	$\sim A, I$ A, I
$\sim A, F$ A, T X	$\sim A, I$ A, I X		

The fourth branch remains open. There is no mixed double negation elimination.

3.6.2 Some versions of classical theorems are true with $[\sim]$ replacing $[\neg]$.

Example: $\vdash (A \wedge B) \supset \sim(\sim A \wedge \sim B)$

and other DeMorgan-principles.

3.6.3 Some crucial non-validities of PC, like *Aristotle's Thesis* $\neg(A \supset \neg A)$, are still not provable in their $[\sim]$ version. If A is indeterminate, $\sim(A \supset \sim A)$ is false.

3.6.4 Given the truth value operators and $[\equiv]$ theorems of the extended language are:

$$\vdash A \vee \sim A \vee IA \quad (\text{Quartus Non Datur})$$

$$\vdash A \equiv \sim\sim A$$

$$\vdash IA \equiv (A \equiv \sim A)$$

Note also: $\vdash A \equiv \neg\neg A$, but $\nvdash A \equiv \neg\neg A$

3.7 Reasoning from both A and $\sim A$ in case of a borderline case cannot establish a conclusion by using Dilemma, and it should not be able to do so. (Supervaluationism having only $[\neg]$ at its disposal misses this point.)

3.8 PCV can be extended to a quantificational logic PCVQ by adding the usual quantifiers with the obvious truth conditions: (i) $[\forall xPx]$ is true if [P] is true of all objects of the domain, false if false of an object in the domain, else indeterminate. (Like in FOL this presupposes a non-empty domain of objects and no non-referring singular terms. The debate about Free Logic should be kept apart from the debate about the logic of vagueness.) (ii) $[\exists xPx]$ is true if [P] is true of an object in the domain, false if false of all objects in the domain, else indeterminate.

3.8.1 PCVQ as quantificational PCV can use the obvious tableaux rules for $[\forall]$ and $[\exists]$, given the truth conditions above. Again the [F or I] cases correspond to the classical $[\neg T]$, and thus PVVQ is a conservative extension of FOL, *coinciding* with FOL in the common language.

3.8.2 Way more difficult is the question whether $[=]$ -sentences (identity statements) can be vague, and whether this implies that there a vague identities or even vague objects. (This debate cannot be taken up here, as the topic of these notes is a justification of a 3-valued approach in the form of PCV in general. The difficulties about identity statements and vague identities concern all approaches to the logic of vagueness alike.)

§4 Sorites, Borderline Cases, and other Issues

4.1 As a 3-valued account involves cut off points the inductive step of Sorites type reasoning is undermined. Sorites type paradox can be avoided. The inductive/general premise is false.

4.1.1 The approach here can reject the Sorites reasoning and explain its supposed 'appeal': From the individual speaker's perspective the Sorites reasoning (appealing to too fine-grained

differences for the speaker's discriminatory abilities) seems seductive, the claim of existing cut off points surprising. Once we realize, however, that cut off points are a statistical regularity their unobserved existence is not surprising. And the community of speakers has no uniform discriminatory abilities, so *collectively* (statistically) it can make a distinction between cases where individual speakers cannot.

4.1.1.1 The Sorites scenario is typically *set up* with respect to an individual and her judgements. For patterns of usage, which are social, this is just the wrong set up. Sorites interview sessions are another example of a philosophical thought experiment where the set up preconfigures the 'natural intuitions. (Cf. §2.3.1 on the problem of a social interview session.)

4.2 Establishing an intermediate range of indeterminate cases without implying that there are single borders for all users establishes a *core* of indetermination – and reduces 'higher order vagueness' to falling somewhere within this zone for some but not all users. The impression of higher-order vagueness (that it may be vague what is a borderline case) stems, in this light, not from an iteration of vagueness which cannot be reduced to simple vagueness. For each user it seems vague what is vague – this is the proper impression of 'higher order vagueness' – but not because vagueness can be non-trivially iterated, but because the individual user cannot recognize the cut off points, as they exist as statistical regularity, but not as observable barriers (imagined like tollgates). All phenomena of higher order vagueness can be dealt with as phenomena of vagueness in the sense of occurring within the indeterminate range. They correspond to individual users' insecurity where the customs of usage (the statistical cut off points) are. There is no need for special logical machinery for 'higher order vagueness'.

4.2.1 In the presence of truth operators the reduction principle $I \mid A \supset I \mid A$ holds trivially, because the antecedent will be always false.

4.3 In the debate about 'penumbral truths' with respect to vagueness much depends on how to understand the constraints put forth by the penumbral truths.

4.3.1 Say two colours are continuous in a colour spectrum and we have a borderline case: C_1 and C_2 (as colour classifications) are both indeterminate. Nonetheless, the argument goes, we want to consider $C_1 \wedge C_2$ as simply false and $C_1 \vee C_2$ as true. It seems, on first sight, that a 3-valued approach fails here, as both are I (by the truth tables), whereas supervaluationism answers correctly. A 3-valued approach like PCV puts a strain on our understanding of penumbral truth. The strain should not be denied. The idea behind the penumbral truths can be captured, and the demands satisfied, however, on a certain reading. Although $C_1 \wedge C_2$ is I , in the presence of $[\neg]$ we have $\neg(C_1 \wedge C_2)$ being true, and this is close enough (the conjunction being not true) to fulfil the penumbral demand (that the conjunction should be false) here. If C_1 and

C_2 are continuous in a colour neighbourhood, and we wish $C_1 \vee C_2$ to be true in that neighbourhood, we can lay down this penumbral demand as $N_C \supset (\neg C_1 \supset C_2)$, i.e. being in that neighbourhood, not being C_1 means being C_2 (and correspondingly in the other direction). Then, for an object in that neighbourhood N_C is true, so with $C_1 \vee \neg C_1$ we arrive at $C_1 \vee C_2$ by PC, and thus respect the penumbral truth. The examples typically given for penumbral truths rest on semantic postulates – not any mysterious intuitions. Once we make these postulates explicit one can expect that the evaluations demanded for some sentences involving vagueness can be delivered. There remains a residual strain (as $C_1 \wedge C_2$ still comes out I), but we can say what we want to say about penumbral truths.

§5 Criticism and Comparison of Approaches

5.1 Supervaluationism prides itself with sticking to classical logic, but it has non-prime disjunctions, and keeps TND, which should be so using $[\neg]$, but opens no logical space for $[\sim]$.

5.1.1 Supervaluationism argues for keeping classical logic. The argument is short, indeed: (i) partial evaluations can be completed (covering all basic sentences bivalently), (ii) classical logical truth is truth in all complete evaluations, thus: the classical logical truths are the logical truths if all partially evaluations have been completed. One should accept this argument, and it should not be surprising – nor very illuminating. Vagueness does not enter the argument. The core of supervaluationism is not about vagueness, but more general.

5.1.2 Supervaluationism can be coupled with a theory of vagueness. Once it tries to mark the distinction between determinate cases and indeterminate cases by introducing corresponding operators $[D]$ and $[I]$ the resulting logic invalidates many classical inferences and theorems. A theory should be able to express the semantic distinctions it draws without yielding too much damage. Whether operators, and which operators, or a distinction between types of negation are more coherent here needs exploration.

5.1.2.1 PCVQ could, in contrast to supervaluationism, define: a predicate $[P]$ is determinate/non-vague if $P(x) \vee \sim P(x)$ is true for all objects of the domain, otherwise it is vague (has indeterminate applications). One could introduce the corresponding operators then, say like: $DP(a) \equiv (\forall x)(P(x) \vee \sim P(x)) \wedge P(a)$, or if that is too strong, in the extended language of PCV with truth operators as: $DA \equiv TA \vee T\sim A$. One might endorse $A \vdash DA$, and with the Deduction Theorem get $\vdash A \supset DA$, but this – straight forwardly read as: ‘If A is true, then it is definitely true’ – does no harm in PCV as conditionals with I antecedents are true (cf. on irrelevant cases §3.2.1).

5.1.2.2 Note that these operators are not introduced in the common language with FOL. They cannot endanger coincidence of PCVQ and FOL in the common language.

5.1.3 Supervaluationism which believes in higher order vagueness makes the range of the precisifications indeterminate! And this range/collection is quantified over in the definition of [Truth]/[Super-Truth]. Nobody *can* check all precisifications – for any sufficiently interesting (part of) language – so the core argument *has to be* about the essence of this precisifications, namely being complete classical evaluations. ‘Base points’, ‘precisifications’ are conceptual wheels that turn idle once premise (i) about completion of partial evaluations is assumed, or borrowed from classical meta-logic.

5.2 If the borderline cases are neither true nor false, and nothing else, one keeps bivalence in doctrine, but one has introduced a 3rd semantic category of evaluation, and one should be able to express the status of the items in that realm directly. It seems thus more appropriate to introduce a 3rd value.

5.3 Vagueness and contradictions should be kept apart. A borderline case neither is judged to be A nor to be $\sim A$. Paradigmatically vagueness shows a lack in coming to a classification. Therefore, to model an instance of vagueness as a contradiction (fulfilling both A and $\sim A$) assumes determinateness – namely overdetermination – where we should see a lack of determinateness. Speakers do not see objects indeterminate with respect to a predicate as contradictory. Our evaluation of indeterminacy may sometimes stem from being torn between seeing something as A or as $\sim A$, the typical result being that we just cannot say, not that we assert both. Paraconsistent treatment should be reserved to those cases where we actually have (in the case of the semantic antinomies) proof of something being both A and $\neg A$ ($\sim A$ typically does not apply there or strengthened antinomies move from $\sim A$ to $\neg A$). Paraconsistent treatments of vagueness (in terms of predicate extensions, and anti-extensions, and their overlap) should be rejected.

5.4 If we evaluate a sentence as I this could happen because we cannot make up our ‘semantic minds’ to ascribe either T or F. Someone may argue: Maybe further reflection or tests lead us to a determined value T or F. From this perspective a value I could be understood as ‘could turn out F or could turn out T’. This makes I an epistemic indetermination, not a semantic one fixed by the language practice. This is not an account of vagueness, but of lacking knowledge. Seeing I this way justifies quite different truth tables! Crucially $I \supset F$ and $I \supset I$ should be evaluated as I. This gives Kleene’s 3-valued tables, where no distinction between $[\neg]$ and $[\sim]$ is to be found. This logic lacks theorems altogether. PCV deals with vagueness, not lack of knowledge.

5.5 A 3-valued logic puts all borderline cases in the same gap, as does supervaluationism. This seems to neglect that in the gap region individual items may be arranged so that some show a better fit than others, are P to a higher degree. This makes degree accounts attractive to some. Degree accounts, however, have several disadvantages (from invalidating key logical principles to the problem that we cannot cognitively introspect fine grained degrees of fit assumptions). It seems more advantageous to avoid both the psychological problem of degree identification and to follow classical truth functional logic as far as possible. That some items or instances seem to fit P better despite being vague instances can be seen as an aspect of vagueness itself. Vagueness means that some items fit a predicate or its strong negation not sufficiently, but vaguely relate to the predicate's instances or counterinstances more or the less. 'More or the less' already expresses that this is a range of vague or borderline cases. Ascribing a degree meant being determinate about it (i.e. a degree of fit), where vagueness is indetermination, thus also indetermination of fit. The criticism levelled at 3-valued approaches can rather be turned into an argument for a 3-valued approach, which considers all vague cases just to be vague.

§6 References

Rosanna Keefe's *Theories of Vagueness* (2000) is an introduction to the key theories of vagueness, and argues itself in favour of supervaluationism. The *Stanford Encyclopaedia of Philosophy* has an entry on vagueness by Roy Sorensen, his book *Vagueness and Contradiction* (2001) presents his version of an epistemic account.

The reader *Vagueness* (1997), edited by Rosanna Keefe and Peter Smith, collects some ground breaking papers introducing the main theories of vagueness. Some newer proposals following these debates can be found in the first half of *Liars and Heaps* (2003), edited by JC Beall.

Further references can be found in these sources.

Ulrich Blau's *Die dreiwertige Logik der Sprache [The 3-valued Logic of Language]* (1978) argues that natural language logic is 3-valued because of vagueness and reference failure. One may argue about other (Free Logic) treatments of reference failure, but agree with Blau on vagueness. In the PVC syntax PCV and Blau's logic J3 coincide (by a trivial substitution of symbols). Blau proves important meta-theorems for this logic. As far as I know, the book has unfortunately never been translated into English.