True Contradictions for the Sober Minded

Manuel Bremer

Dialetheism is the thesis that there are true contradictions: sentences α so that

$$\alpha, \neg\alpha, \alpha \wedge \neg\alpha, \alpha \equiv \neg\alpha$$

are true. Paraconsistent logics are logics which can apply inferential rules in the presence of true contradictions without trivializing the consequence set (i.e. not making all sentences true/derivable by *ex contradictione quodlibet*).[1]

Many philosophers find the idea of true contradictions outrageous and bizarre. It sounds like an idea that is barely comprehensible: How could ever α and $\neg\alpha$ (for some not too deviant negation "$\neg$") be true? Moving to objects: How could an object have contradictory properties (for some not too deviant and anti-realist account of properties)? Consistency is one of the main building blocks of coherence in reasoning – whatever else may go into coherence. How could one ever argue or state a claim when one foregoes consistency?

These concerns have been raised at least since book Γ of Aristotle's *Metaphysics*. And proponents of inconsistency or true contradictions have been working on providing answers or debunking the seriousness of the problems. The debate will not be retold here. Instead a lit of issues and theses is collected to outline a position on true contradictions that could find the approval of Logicists, Logical Empiricists, and Analytic Philosophers in general.[2]

---

[1] Not all proponents of a paraconsistent logic agree on moving from $\alpha \equiv \neg\alpha$ to explicit contradiction $\alpha \wedge \neg\alpha$, as this depends on *tertium non datur* and dropping the meta-/object-language distinction, e.g. Ross Brady in *Universal Logic* (Stanford, 2006). I presuppose some general knowledge of paraconsistent logics and focus on the philosophical idea of dialetheism. On the paradigmatic arguments for dialetheism see the *opus classicus*: Graham Priest's *In Contradiction* (2nd Ed. Oxford, 2006), where also the systems LP and the adaptive logic Minimal Inconsistent LP (MILP) as well as a discussion of the assertion/rejection distinction can be found. On different approaches within paraconsistency and several paraconsistent logics see: Manuel Bremer, *An Introduction to Paraconsistent Logics* (Frankfurt/Bern/New York, 2005), where also the adaptive logic UL4 with bivalent truth value operators and restrictions on *Modus Ponens* and *Substitution of Identicals* can be found; cf. Manuel Bremer, "Believing and Asserting Contradictions", *Logique et Analyse*, 2007.

[2] I fancy that someone as open-minded as Rudolf Carnap could endorse dialetheism of the type set out here.

1. Paraconsistency and Dialetheism ('PD' for short) are not an expression of 'getting soft' on logic or taking some esoteric view of philosophy and the world at large, quite to the contrary. PD aims to extend logic and the logical treatment of philosophical issues to new areas. Inconsistency has been anathema to logic because not just the Law of Non-Contradiction (LNC), but because of *ex contradictione quodlibet* (ECQ or 'Explosion'). PD provides a framework to deal with such formerly excluded issues. PD does this without all sentences being true ('Triviality' for short).

2. One may consider antinomies like the Liar as abnormalities of semantics or other fields which show antinomies. As abnormalities they are set aside and one deals with those questions treatable by science. This is an absolutely acceptable stance on the antinomies. It mirrors the attitude to anomalies in science, broadly taken, say in physics. One acknowledges that a problem persistently shows up but isolates it by not dealing with it or its connections to the treatable issues. One waits for some more comprehensive theory to come that will include or clear away those anomalies. Every enlightened realist will assume that reality can be dealt with in science, even if in some remote future theory. So, we may ignore the antinomies and mind other philosophical business or we aim at a more comprehensive theory.

PD claims that *now* is the time to deal with the antinomies. If philosophical problems of PD amass to an unbearable extend (e.g. alleged problems of generating too many new antinomies and coming with a bizarre metaphysics) then the position of treating or isolating the antinomies as anomalies is a default scientific fallback position.

3. What is not acceptable is a position of dealing with the antinomies – supposedly 'solving' them – that lands oneself in an even more obscure position than the antinomies themselves or PD. Such is the case with (semantic) hierarchies as solutions to the semantic antinomies. Hierarchies taken literally are ineffable (i.e. cannot be taken literally) as there is no way to talk about the totality of the hierarchy – from a non-existing outside of the hierarchy. They are not just contradictory in denying universal semantic talk and being just that, they are even, therefore, performative mysteries: they state that something cannot be done and do it at the same time!

Performative inconsistency is too much – even for PD. What one does one's theory should allow as possible and achievable. Ineffability would be the end for just the theories philosophy aims at: universal theories of language (including all of semantics) and universal theories of knowledge.

4. PD is committed to philosophical universal theories. This involves semantic and epistemic closure. The language one's theory is expressed in and the language the theorist just uses belong to the field of study the theory covers. Natural languages and reflexive epistemic reasoning are closed in this sense. For them taken as a whole the distinction between object- and meta-language cannot apply. The meta-logic of PD has to be the preferred paraconsistent logic itself.

5. As a paraconsistent logic is more restrictive than FOL – and other 'explosive' logics like Intuitionistic Logic – the restrictions will curtail certain forms of inference: directly ECQ and indirectly all theorems or rules that lead to Triviality. Different paraconsistent logics follow different restrictions: some put them on Contraction and structural rules, properties of negation or properties of conditionals. MILP, UL4 and other adaptive logics restrict the use of *Modus Ponens* (MP/$\supset$E) to consistent detachment. UL4, furthermore, restricts Substitution of Identicals (SI/=E) to consistent objects.[3]

The aim in all camps of PD is to allow for 'classical recapture': to apply FOL (or some other non-paraconsistent logic) in all the consistent contexts, to derive theorems that given a more restrictive logic were not derivable. Inconsistency calls for local measures and not for dropping achievements of standard mathematics and logic.

Adaptive logics treat consistency as the default situation and adapt to the occurrence of inconsistency. If some statement or object turns out to be contradictory consequences drawn by directly using these (either as antecedent in MP or in a case of SI) have to be retracted. Universal

[3] An object or singular term γ denoting an object can be classified as 'inconsistent' if for some sentential function δ we have $\delta(\gamma)$ and $\neg\delta(\gamma)$. Contradictory sentences themselves are inconsistent objects with respect to some semantic evaluations. Both (=E) as well as Substitution of Equivalents have to be restricted or given up in paraconsistent logics.

inference (that operates under the danger of inconsistencies involved) states relative derivability with respect to premises and a set of presuppositions of consistency (noted, e.g., in UL4-derivations). The general attitude behind *this* strategy can be stated as:

(LNC*) Do not draw direct inferences from inconsistent sentences or objects.

This strategy acknowledges that inconsistencies are abnormalities in reasoning. Indirectly we draw conclusions from their occurrence, Dialetheism itself arises as a theory of them. But we should not directly (in immediate inferences like MP or SI) use them, as this leads just to more inconsistency and confusion. PD of *this* type does not revel in the exploration of inconsistencies. They are remarkable as features of our universal semantic reasoning, but a nuisance at the same time. They need not be cherished. One should not – as unfortunately some proponents of PD have done and do – erect a world view based on them, quickly declining into esoteric metaphysics (like 'noneism') and (postmodern) mumbo jumbo.

6. It may even be that some of the restrictions used in logics like MILP or UL4 *block* the well-known proofs of the semantic antinomies – but that can hardly be held against dialetheism. Dialetheism can be weakened to the thesis that *if* given some basic principles of truth, denotation, and (semantic) closure we derive at contradictions, these may be taken as being true. If some well-known examples are lost that does not matter. The purpose of PD should not be to have true contradictions, but to have semantic closure or naïve set theory or … *even if* this involves accepting some true contradictions. The controversy has centered on the dialetheist's claim that there are true contradictions, but the starting point has always been some other philosophical tenet. So, in case there are no true contradictions, so the better for semantic closure – or naïve set theory if one wants to have Naïve Comprehension. I take the philosophical point of dialetheism for the sober minded to be that *even if* there are true contradictions this is the price to pay in some universal theory and logic. So, we better model universal logic, reasoning, belief and assertion on the assumption that there *are* true contradictions. I assume that some semantic antinomy (still) can be proven and thus is taken as true by a dialetheist [see No. 7].[4]

[4] It should also be mentioned that those systems of paraconsistent logic like UL4 have sentences that look like Strengthened Liars (e.g. sentences saying of themselves that they are *false only*). Switching

7. Strict Dialetheism claims that there are true contradictions by exhibiting some. Some antinomy α is a true contradiction because we have a proof of α and a proof of $\neg\alpha$.

For an approach like using UL4 this constructive argument may not be available. The standard reasoning for the Liar does not go through because of restrictions on MP. The standard reasoning for 'Berry's Paradox' (of the least number not denoted by phrase of less than 100 letters) does not go through because of restrictions on SI. Nonetheless, one may be a 'Non-Constructive Dialetheist' – just like one should be non-constructivist in many areas apart from computability. The reason for claiming the existence of true contradictions for a Non-Constructive Dialetheist is an induction on attempts to solve different kinds of paradox/antinomies. Looking at several attempts to solve the semantic antinomies (especially those referring to truth value gaps or semantic hierarchies) one can claim:

(FORK) Attempts to circumvent the semantic antinomies use conceptual resources which are either ineffable, thus providing new performative antinomies, or lead to strengthened forms of the antinomies.

'Revenge' versions of the Liar (targeting truth value gap approaches) are typical examples of this dilemma. The thesis (FORK) can be understood as the result of a *double induction* on the number of attempted solutions and the number of paradoxes/antinomies. If this induction can be undercut (e.g. by some final solution to the semantic antinomies) then there *are no* true contradictions, dialetheism is false. Supporters of consistency will shed no tears. Supporters of universal philosophical theories will be content as well: it was the achievement of universal theories involving closure that mattered, not the exhibition of inconsistencies. That our semantic and epistemic faculties turn out consistent in such a scenario is the new remarkable insight then, backed up – still presuming the new consistent universal theory – by an account why the appearance of antinomies showed up (vaguely similar to Kant's dissolving of the antinomies of pure reason).

to evaluation relations and the restrictions on rules like (MP) and (SI) in proofs, however, avoids getting hyper-contradictions. The only interesting observation with respect to these Strengthened Liars is that they seem to be incapable of achieving what they assert of themselves (i.e. being false only).

8. The semantic antinomies are the only ones that cry out for a PD treatment. PD need not extend to the objects of empirical sciences, to issues of vagueness, and not even to set theory. The minimal dialetheist commitment can thus be understood as: Non-Constructive Semantic Dialetheism (NCSD). Other areas may be treated by a paraconsistent logic, but this either can be done with a more instrumentalist attitude or this treatment is presumably convenient but not required as other means to deal with these areas are open.[5]

A fictionalist about set theory, and thus mathematics in general, has to account for the partial applicability of mathematics in science, and the role of the non-applied parts of set theory and mathematics as mere scaffolding for the applicable parts.[6] ZFC presents a story about the *sui generis* realm of the iterative hierarchy (a.k.a. the set theoretic universe V). Within this story (e.g. by Reflection Principles) set theoretic modelling is explicable given the postulates concerning the special entity V (being neither an ordinary object in a domain, nor a set or any collection of a type which itself can be collected).

Naïve Comprehension is acceptable once we think of non-abstract objects. The calculus of classes contains both the idea of Abstraction $[x \in \{y|\varphi(y)\} \equiv \varphi(x)]$ as well as a universal class and absolute complements. Only if we consider all objects (including sets themselves, if we believe in their existence) the antinomies result. It is an insight that Naïve Comprehension does not hold in this wider realm, only Separation and the rest of ZFC do, taken as a story.

9. NCSD need not be paradoxical itself. Given the resources of either adaptive logic and/or a distinction between assertion of a negation and denial, and/or bivalent truth operators in a logic with a corresponding paraconsistent and relevant consequence relation (like UL4) saying of a true contradiction that it is true need not be a paradox or true contradiction itself. A philosophical theory should be just true, how amusing the spread of paradox may sound to some ears. NCSD may not even have a single specific true contradiction to show (off).

The field of inconsistencies seems to invite all kinds of speculative and dialectic maneuvers alien to – what we may call in analogy to other fields – 'the working analytic philosopher', if there are

[5] Details are, again, skipped here. There exists an extensive literature on vagueness, supervaluations etc.

[6] Without going into the details of such accounts I assume that this can be done, and has been done.

such. PD may have not the best reputation not just because of the outrageous claim of true contradictions, but because of what has been made of that in philosophical reflection and because of extending the idea of true contradictions to other fields (even to ordinary objects and, of course, quantum physics).

This need not be so with NCSD. The only true contradictions to assume are sentences of semantics (most likely involving forms of self-reference). The only contradictory properties are semantic properties. True contradictions if they live anywhere, they live in the realm of meta-representational vocabulary and properties.

10. NCSD can be an approach to the hard problem of reconciling our attempts at universal theories with the seemingly robust occurrence of antinomies in meta-representational concepts and reasoning. As we want to tackle these areas we have to deal with their abnormalities. NCSD *allows* to do this in a way that stands in the best tradition of Logical Empiricism and Analytic Philosophy.

Version 1, 2024.