

Lectures on Universal Logic

Lecture 7 – The Problem of TND in Universal Logic

One point of disagreement between Routley's and Brady's universal logic programmes is the validity of TND. Routley endorses TND, and employs it to derive explicit contradictions from the paradoxes. Brady denies TND – outside classical recapture – indirectly for the reason to avoid just these derivations.

In the light of the strong universal logic programme, mentioned in Lecture 1, this poses the question whether our natural understanding of negation, our use of negation in natural (language) reasoning endorses TND.

Is there room for **truth value gaps** in (strong) universalist reasoning? I don't think so.

There is something wrong with the concept of truth value gaps when applied to natural languages.

Many 3-valued approaches to paradox and antinomies can be rejected as insufficient, because they face strengthened versions of the antinomies, or as linguistically limited (in the sense of relying on further linguistic resources), because they employ distinctions in semantics which they cannot express themselves. They are applicable only to partial languages and not to inclusive languages like a natural language, which has to have the resources to express any semantic distinction, as the arguments introducing these distinctions are expressed in natural language or a canonical regimentation of it. So many such gap theories fall prey to 'Strengthened Liars' and 'Revenge', which reintroduce the bifurcation needed for the antinomic reasoning.

Pragmatic approaches to the antinomies often suffer from their proximity to such 3-valued approaches.

[Note on usage: an 'evaluation' assigns a truth value, an 'interpretation' assigns meaning, so that there may be an interpretation without an evaluation.]

'Real Gaps' would be gaps that do not result from assigning a third value (like 'undecided'...), but result from abstaining from interpreting a sentence **altogether**. The crucial

idea to be tested rests in the assumption of a meta-semantic rule which can enforce the retraction of a supposed evaluation of an antinomic sentence.

Plan: One has to start with a preliminary discussion of the existence of token-reflexive truth value bearers in ordinary language. The interpretation of statements made by the use of token-reflexive sentences invites a meta-rule for truth value gaps. The rule states that whenever a sentence A on a supposed interpretation leads to a contradictory evaluation, A has no evaluation. Applying the rule and delineating those statements, however, raises the issue of talking about their (absent) interpretation. This can be expanded into an argument for the inexpressibility of real gaps in natural language.

Applying the (pragmatic) distinction between a sentence and its use in a statement, which has a content, one can always come up with a sentence which, although syntactically not identical to the antinomic sentence, can be used to make a statement with the same content as the antinomic sentence. This in itself is not surprising. The failure of the meta-rule (and thus corresponding pragmatic theories of real gaps) rests in the fact that the resources needed to use the meta-rule fall prey to this. Sentences about occurring gaps should not be without evaluation themselves.

Token-Reflexive Sentences and Statements in Ordinary Language

The word ‘this’ can be used as a deictic expression roughly synonymous with ‘that’ or as a strictly reflexive indexical. So

(1) This sentence has five words.

can mean – in one reading to be disambiguated –

(2) That sentence (over there) has five words.

or it can mean – in another reading to be disambiguated –

(3) This (very) sentence (I am using right now) has five words.

In expositions of the antinomies one often uses ‘this’ in the sense of (3), e.g.

(4) This statement is not true.

As the ‘this’ is **strictly reflexive** its reference does *not* vary from occasion to occasion (of using sentence (4)). It always refers to (4).

Statements result when a speaker uses a sentence on an occasion. Sentences often are indexical and thus are neither true nor false as they stand. The indexicals have to be anchored to a situation of usage. *What is said* on that occasion then can be true or false. The distinction has found use not only in pragmatics, but is part *inter alia* of the framework of situation semantics. **Statements are eternal.** Statements can be presented by *eternal sentences* or by some abstract representation (like ‘infons’ in situation semantics or state of affairs in other ontological frameworks).

In these abstract representations any indexical part present in the sentence used (say “I sit on that chair”) is replaced by the item/referent talked about in the statement: “I” is replaced by the speaker, “sit on” by the relation of sitting on, “that chair” by that chair, including times and places, and so forth.

Therefore: An occasion of using a Liar sentence gives rise to a Liar *statement*, which is eternal, and refers to this very statement, and the property of truth, and so forth. For the statement (i.e. this objective content, which could also be specified as a tuple of items including that very statement) it is of no importance how the items are referred to.

[As statements can at least be expressed by eternal sentences I follow the practice of speaking of (eternal) sentences as truth value bearers. And I take eternal sentences as expressing the *content* conveyed by using some sentence in an assertion (in a situation).]

Thus, when someone reasons about (4) that (4) has no semantic value, and thus is not true, he *is not making another statement* than (4) does (or somebody using (4) does). He can speak French or whistle, as long as some conventions of fixing reference are in place he is **making the same statement**. Of course, his speech act takes place on another occasion than mine, two tokens of sentence (4) are used. But he makes the same statement that anybody makes who uses (4). The “this” in (4) does not indexically refer to some sentence used in a situation of usage, but refers strictly reflexive to (4). And as his reasoning leads back to the statement made by (4), the antinomic reasoning proceeds as it always does. One may be misled by the usage of ‘this’ or other natural language renderings of the antinomies, but shortcomings of phrasing the antinomies in these ways should be surmounted by formalization.

Some of the problems concerning whether some sentences really ‘say’ something or not might be solved by trying to be more precise on the formal side of the sentence/statement distinction. Unless one believes that natural languages are beyond formal treatment there have to be some formal means or representations which capture that distinction. Logicians exploring the logic of assertion and rejection sometimes use the syntactic derivability sign

(i.e. \vdash) or some new device (like: \Vdash) to capture assertive force. Assertive force can be represented in natural languages themselves, which, of course, pretty nicely explains that we can write papers about it.

Now, as the sign of assertive force might define what distinguishes a statement from other ways of using sentences, and as this operator/sign is part of the formal machinery of language, one can distinguish well-formed formulas containing it at the proper place from others. Thus, we can define the class of statements (or statement representing eternal sentences). As this is a syntactic identification, the process of doing so is primitive recursive. Therefore, we have all reasons to believe that the antecedent conditions of the *Diagonal Lemma* are fulfilled, and thus *that there is some sentence λ equivalent to the sentence that asserting λ results in a false statement*. As no indexicals are involved that sentences λ **not accidentally** is usually taken as the eternal sentence representing the Liar (statement). Once we have that eternal sentence in the context of an assertion sign it seems to be pretty *ad hoc* to deny that it says what it says. Even if no one bothers to utter it right now someone may do so, and λ just has the objective content it eternally has.

There are, thus, token-reflexive sentences in ordinary language. Even if “this” in (1) usually refers to another sentence (has an unspecified argument position), we can substitute a name of the diagonal of that sentential function into that position thus arriving at a token-reflexive sentence. From a syntactic point of view **ordinary language seems to contain token-reflexive sentences**. Otherwise ordinary language wouldn't be as expressive as some artificial languages.

A Semantic Rule for Gaps in Ordinary Language

How are we to evaluate (4) then? Given standard semantics we arrive at the antinomy. – Can we avoid evaluating it at all? Even if we cannot make sensible use of (4) [see below], can we suspend (4) from having an evaluation? Given mildly realistic assumptions in semantics, even if we personally do not care about the sentence it has an evaluation, thus ordinary language will be inconsistent, whether we care or not.

One might resort to some meta-rule on evaluations that revises all evaluations in case they land us in contradictions.

Firstly, such a rule will be beyond standard logic towards default or adaptive logics (cf. Lecture 10).

Secondly, **if we do not distinguish object-language from meta-language this rule and its negation may enter into reflexive constructions**. In contrast to a sentence, however, if there was a corresponding reflexive rule generating contradictions, we simply need not adopt it. A sentence saying of itself that it was not subject to the original rule might be without evaluation itself.

The meta-rule had to say something like

(MR)

Whenever a sentence *A* on a supposed interpretation and evaluation (i.e. a straight forward application of the semantic rules) leads to a contradictory evaluation, *A* has no evaluation. The supposed evaluation is rejected.

The rule enforces and says in effect that we will never accept contradictory evaluations. It relativizes the coverage of our semantic rules, which are otherwise understood as applying to all sentences. It is a rule about semantic rules. Given (MR) sentence (4) in fact has no evaluation. Relatives of (4) which token-reflexively say of themselves that they have an evaluation and that they are false are not simply false (as this would be an evaluation), but lack an evaluation themselves. (MR) regulates the absence of interpretations in general, even involving sentences referring (indirectly) to (MR), like

(4') If this has an evaluation, it is false.

(4'') This has an evaluation and is false.

Such sentences are without evaluations themselves. (4'') is antinomic if evaluated, so one just leaves it without evaluation.

“This has no evaluation” can be consistently interpreted as false. So, *it seems* that (MR) allows for consistent self-regulation and does not face a Strengthened Liar, so far.

Do sentences like (4) have a use in ordinary language?

In indirect speech we can, of course, mention them as examples of problematic sentences. In a straight forward assertive utterance the felicity conditions of such an utterance cannot be met, given (MR), as we assert as true a sentence which cannot be evaluated. In a liberalized understanding of the felicity conditions we may even allow for such utterances inasmuch as they succeed in putting forth a sentence to be evaluated, but are followed then by an act of retracting the assertion. The speaker might be blamed for violating the rule of asserting only

what one believes to be true, or be blamed for believing this sentence to be true (i.e. for incomplete reasoning with respect to the sentence or neglect of [MR]).

In any case, showing **that we can achieve no purpose by using (4)**, or even showing that an attempted assertive utterance will misfire, **does not by itself entail that (4) has no evaluation**. We need the additional appeal to a meta-rule like (MR). In this respect the treatment of the antinomies in ordinary language explored here is *not* just pragmatic. It concerns our framework of semantic rules.

Further on, that (4) has no assertive use and receives no evaluation need not and should not entail that (4) has no interpretation, no meaning. We understand what (4) wants to say, because we employ our capacity to understand token-reflexive sentences. Many of them are harmlessly true, like “This is a token-reflexive sentence”.

We understand (4) inasmuch as we know which semantic rules are to be applied to it (compositionally). We derive an interpretation. Once we arrive at the supposed contradiction (MR) will be activated and no evaluation is delivered on (4). The real gaps are gaps with respect to the evaluation of sentences, they are not semantic gaps, say in the sense that some sentences have no meaning.

This whole approach looks like a semantics with truth value gaps (or even interpretation gaps), and then a **Strengthened Liar**

(5) This is not true.

should land us in a new version of the old antinomy, because *being* not true by falling into the *gap category* seems to make (5) true, and we have regained an antinomy. Having *no* evaluation cannot be just another evaluation (included in the opposite of being true), then, without contradiction. Gaps must be *real gaps*. They have to lead to real gaps in evaluating discourse and to real gaps in drawing inferences. The semantically crucial issue is distinguishing the statement that (4) is not true, as it has no evaluation, from an evaluation. So far, we have no theory how such 'real gaps' should work. Suppose, for the moment, that gaps do not fall prey to a Strengthened Liar. Nonetheless **we have to be able to talk about them**.

The Inexpressibility Argument

One way one should not try to circumvent a Strengthened Liar is by restricting “not” in (5) or related sentences to be some kind of 'inner' negation which carries us only from truth to

falsity and from falsity to truth. This way **we could not even express** that sentences like (4), which have no evaluation, are therefore *not* contradictory (i.e. true and false at the same time). To express the theory of gaps and (MR) we cannot forsake a 'wide' negation.

Meta-linguistic sentences talking of the predicate “() is true” also do not change the picture, as the old antinomies can easily be transferred into meta-linguistic versions like

(5') The predicate “() is true” cannot be successfully applied to this very sentence.

which raise the same issues of contradictions or inexpressible semantic facts.

We face **problems of inexpressibility** with our treatment of (5).

Saying of (5) that it has no evaluation should be true. Suppose it is. **Then it is also true that (5) is not true** (as being 'true' would be an evaluation). And it would be a limit of semantic expressibility if we could not say so. Suppose we could say so. Does *this* then *imply* that (5) is true after all? – No! No, because to derive this claim we have to apply some semantic rule for “() is true” to (5), the application of which will be retracted by (MR). How can we say then that (5) is not true? *Saying* that (5) is not true seems like an instance of the type of (5), at least we seem to make the same statement like (5). That saying of (5) that it is not true has meaning (an interpretation) seems to be not enough, since we seem to know that it *is* also true. Saying of (5) that it is not true should be **evaluated** as being true, but this brings us full circle to evaluating a statement having the same content as stating (5).

The only option seems to consist in a better understanding of what (5) really says given the presence of (MR) and real gaps in evaluations. Either (5) says the same as (4), namely says of itself that it has the semantic property of being false, and so we have dealt with it already, or we have

(5'') This has no evaluation.

which is *just false*, since assuming it to be true leads to contradiction. So we may reason that either (5) has no evaluation (by being [4] in disguise) or (5) is false (by being [5'']). Saying of (5) that it has no evaluation is (5''), which is false. So if (5) *has* no evaluation we cannot say so truly. So barring inexpressibility we should say that (5) is false:

(6) (5) is false.

but if (6) is true, it **entails**

(7) (5) is not true.

which again has the same content as (5). Our sentence (7) about (5) not being true is, of course, not the token-reflexive (5). (7) differs in logical structure, *but not in content*.

And thus, given our reasoning about (5), (7) is false, and again we have not succeeded in saying truly of (5) that is false. Thus, we arrive at *inexpressibility* after all, whatever may be the semantic matter concerning (5). Of course, this is disastrous, as *sentences about occurring gaps should not be without evaluation themselves. Especially the claims of the gap theory should be true simpliciter. Semantic properties should be expressible in ordinary language.*

(7) does not *assign* an evaluation to (5), and thus the *desideratum* for a theory of real gaps mentioned is *fulfilled*, but even abstaining from assigning an evaluation to (5) it expresses a semantic property of (5), which *prima facie* entails facts about evaluations. The theory of real gaps thus lands in deep trouble.

Opening Gaps by Pragmatics?

The *Principle of Expressibility* in speech act theory states that everything that can be meant can also be said. It licenses the move from performing an illocutionary act (e.g. asserting “The cat is on the mat”) to expressing the illocutionary act by an illocutionary force indicator (e.g. “I assert that the cat is on the mat”). The *Principle of Expressibility* thus entails that there is *no strong pragmatic/semantic-divide* in the sense that pragmatics allows us to do something with language which cannot be expressed into a sentence. Performing an illocutionary act is an intentional action, and the intention should be expressible. Reflecting which illocutionary act to perform I may deliberate on my evidence, my intentions and consult my knowledge of the felicity conditions of some speech acts. All this requires that I *represent* my intentions, beliefs and judgements (i.e. that they are expressible).

Some authors have suggested introducing or *admitting a special speech act of rejection* distinct from asserting the negation of a claim in question.

This speech act may serve purposes of very special situations where we (i) either want to reject both p and $\neg p$ as *inappropriate* or (ii) want to express that, although for us dialetheists sometimes contradictions have to be asserted, in this case we *only* assert p and reject $\neg p$.

These are special context which emphasize non-commitment or *unique* commitment.

Nonetheless, in both cases we should be able to express what we want to achieve by such acts of rejection. In the non-commitment case we want to express that both p and $\neg p$ are not true [resembling the situation just discussed above] or simply evaluate both as ‘indeterminate’ (this being a further truth value and no real gap in evaluations). All this is expressible. In the

unique commitment case the dialetheist has to make use of further negation junctors or truth operators which allow her to express the claim that the rejected sentence is false *only*. Again, **rejecting a claim becomes expressible**, and rejection expressed in that way may be different from asserting the opposite using standard negation.

Natural language allows for a plenty of illocutionary forces, but all can be expressed into illocutionary force indicators with a clear semantics. **Thus, real gaps cannot be introduced by moving from semantics to pragmatics.**

Universality Again

What have we seen? – Even leaving Strengthened Liars to the side and exploiting the presumably universally applicable meta-semantic resources of ordinary language coupled with the resources of a logic dealing with defaults or retractions a gap approach violates the idea that ordinary language can express all semantic facts (crucially including those the gap theory wants to talk about).

The task of universal linguistic philosophy has to be to identify the **features of the universal framework of language**. That framework might not be extensive, and it might be quite formal or parameter ridden what is universal in this sense, but some such framework has to be there. Therefore, **universal linguistic philosophy cannot restrict itself to non-universal languages**.

The language of such a philosophy has to be semantically closed (dealing with its own semantics), thus – at least *prima facie* – engendering semantic antinomies! Philosophers more often than otherwise do not want to deal only with the structure or conditions of talking in some specific language or languages of some kind, but aims at a theory of the basic structures and conditions of having a language *in general*. This requires the corresponding logical and linguistic resources to express the universal claims. There may not be a hierarchy of languages so that we always talk in a *last* meta-language the semantic properties of which cannot be made clear, except in a further turn of the screw (a new meta-language ...).

Universal theories of meaning, truth, knowledge etc. were not to have if we can talk only from some meta-language ‘down’ to some distinct object-language. But these are the very theories that philosophy is after. And notwithstanding their lip-service to hierarchy solutions of the antinomies most philosophers propose their *general* theories of meaning, truth, belief, reference, knowledge etc. They are right to do the latter, since we have such universal concepts. We can investigate and formalize the logical structures of any natural languages. That is one of the central tenets of logic and formalization. Often standard first order logic

and set theory are taken as the meta-language to prove theorems about the logic in question; sometimes – as it should be in intuitionism or dialetheism – the meta-language is taken to be the same logical language as the one introduced or explained; but in all cases the logic and its formalisms are argued for in natural language texts. Natural language turns out to be *the last* meta-language, that meta-language in which the most basic formalisms of some other meta-language were introduced. And natural language turns out to be *the universal* meta-language in that all the formal constructions and sentences of some new system can be translated (read) as ordinary sentences with some formal regimentation. **There is no extraordinary special or deviant new logic which can say something that we cannot say in (some) natural language.** Natural language thus contains the capacity to interpret all these systems. Therefore, the issue of gaps is most pressing concerning natural language. Whatever can be reasoned about gaps can be expressed in natural language. What structures are responsible for this may be the task of advanced philosophy to find out. Without semantic closure we would not be *able* to elucidate a concept that we seem to have! Corresponding to this universal scope of its investigations this philosophy needs the logical means to speak universally. It needs a universal (and thus paraconsistent) logic. A truly universal (paraconsistent) logic can be employed everywhere, supposedly containing a way to distinguish consistent from inconsistent contexts, without loss of proper logical power in comparison to standard logic (like in Adaptive Logics).

Dialetheism can be qualified in that context in two ways: (i) the primary concern is semantic closure and not the assurance that there are true contradictions, i.e. if we can have semantic closure and universality without contradictions we are even the better off, as there is nothing inherently valuable about true contradictions, they are a nuisance the dialetheist is ready to accept only because we cannot give up on semantic closure and universality, (ii) the contradictions accepted pertain to our meta-representational skills (i.e. they affect our concepts of truth, denotation or concept application) and result in contradictory evaluations of claims *about* representations, there are no true contradictions with respect to (first-order) physical properties of space-time regions.