## Lectures on Universal Logic

## Lecture 4 – Brady's Universal Logic Programme

Ross Brady, a Relevant Logics theorist and a co-worker of Richard Routley, has developed a Universal Logic approach over many years. These lectures focus of the state of his theory in his paradigm book *Universal Logic* (2006).

• Brady's Universal Logic is a logic of entailment (as the main conditional). The semantics of entailment is based on a theory of meaning. So one part of this Universal Logic is to capture the proper logic of a relevant conditional, as it, supposedly, is at work also in natural language.

• The logic is also universal in the sense of the previous lectures in that one off its main tenets is to capture naive semantics and set/class theory despite of the paradoxes.

[We look at his theory and his treatment of the paradoxes in Lecture 8.]

• Classic FOL reasoning might be recaptured in classical contexts. The formalization of Brady's logic distinguishes between sentences (p, q ...) and 'classical sentences' (p', q' ...). Thus, in setting up a theory or an argument we have to know beforehand, which sentences are classical, as these allow for classical inferences – in terms of truth preservation – otherwise not valid in the Universal Logic.

• The meta-theory of Brady's Universal Logic is classical! Brady does not follow the 'strong universalist programme', mentioned in Lecture 1, of semantic closure and dropping the distinction between object-logic and meta-logic. Within this classical meta-theory Brady shows that his class/set theory is not trivial (i.e. is absolute consistent).

*Criticism of Classical Logic*

• Brady's criticism of classical PC starts with the paradoxes of Material Implication. Whereas the following sentences may be acceptable for a truth preserving conditional, they are rather anomalous if the conditional is to express a stronger connection, as our ordinary conditional seems to do.

$$p \supset (q \supset p)$$

$$\neg p \supset (p \supset q)$$

$$(p \supset q) \vee (q \supset p)$$

$$(p \supset q) \wedge (r \supset s) \supset (p \supset s) \vee (r \supset q)$$

$$(p \wedge q \supset r) \supset (p \supset r) \vee (q \supset r)$$

For instance, in the last case, a conjunctive antecedent should express that both conjuncts are needed for the consequence. The truth table for [$\supset$], however, makes this a logical truth. The logic of [$\supset$] thus does not allow to read valid principles of conditional reasoning of the logical truths.

• The conditional of classical modal logics ($\Box(p \supset q)$ or ($p \multimap q$)) inherits the main paradoxes of implication like the first two.

• To capture conditional inferences a better conditional connective has to be incorporated into our logic.

• Brady, nevertheless, does *not* primarily aim at a theory of the 'ordinary' conditional of natural discourse with all its (e.g. pragmatic) intricacies and subfields (like theories of counterfactuals), but at a theory of the proper conditional for mathematical and logical theories. Despite these theories often being considered extensional they need a stronger conditional than [$\supset$], namely entailment [$\rightarrow$]. The semantics for [$\rightarrow$], however, is founded on reflections how "to formalize natural language logical concepts" (34) in the context of a theory of meaning relations (see below).

• The Relevance Condition (of sharing a propositional letter between antecedent and consequent) is *a step* towards securing a content connection expressed by a conditional. Entailment goes beyond this condition: the theorems for [$\rightarrow$] should also capture our intuitive ideas of valid theorems of our respective logical theories, i.e. the logic of [$\rightarrow$] should not be too weak. Universal Logic should be suitable for logical practice.

[Brady favours the logic DJdQ, which we look at in following Lectures 5 & 6.]

*Criticism of Relevant Logic*

• Although Brady endorses the Relevance Condition and his logic is (in the terminological sense of avoiding the irrelevant theorems of Classical Logic) a Relevant Logic, he criticises some uses and theories of Relevant Logic.

• As seen above, the Relevance Condition by itself is too weak to ensure a strong enough logic (e.g. containing transitivity principles: if A, B in $A \rightarrow B$ and B, C in $B \rightarrow C$ have letters in common that does not ensure that A and C in $A \rightarrow C$ have a letter in common). One has to look for a logic strong enough, but still relevant.

• Naive set theory raises some problems of relevance for the Axiom of Extensionality (expressed with [$\rightarrow$]) and substitution of identical (as Routley observed, cf. Lecture 3). But this is one of the logical theories to be captured. One thus cannot just – in confidence of [$\rightarrow$] – replace all [$\supset$] in the theories concerned by [$\rightarrow$] and think this will work out fine.

• Most crucially one needs a principled semantic foundation for one's theory of entailment [$\rightarrow$]. Brady aims to deliver this with his theory of meaning containment. The basic idea is that the meaning of the consequent has to be contained in the meaning of the antecedent.

This reliance on meaning does not require a full theory what the essence of meaning is, but a theory of meaning relations, what defines 'meaning containment', whatever meaning is itself.

Meaning, thereby, is one of the central and basic concepts of Brady's Universal Logic. The second central semantic concept, of course, is 'truth'. And truth relations allow to recapture classical reasoning in terms of [$\supset$].

• In comparison to some Relevant Logic like the system R, the semantics of meaning containment may lead to a – comparatively – weaker logic (i.e. some theorems of R are invalidated).

• Some Relevant Logic systems have extra connectives (like an intensional conjunction called "fusion"). DJdQ allows the addition of some of these (e.g. of fusion defined as 'cotenability' of two statements: $\neg(A \rightarrow \neg B)$), but they have a use only as a shorthand for entailment sentences.

*Negation and Tertium Non Datur*

• Like Routley's Ultralogic Brady's Universal Logic does not – except in contexts of classical recapture – contain Classical ('Boolean') Negation (~A being true iff A is not). [¬] in Universal Logic is different from [~]. Classical Negation leads to paradoxes of conditionals like: $A \rightarrow B \vee {\sim}B$

As these are to be avoided so is the general presence of Boolean Negation, although in some contexts [~] has a use.

• Centrally Brady's Universal Logic does not contain TND, in contrast to Routley's Ultralogic (although TND there is phrased not in terms of Boolean Negation), except, again, in contexts of classical recapture. In classical contexts: $A \vee {\sim}A$

TND is rejected by a reflection on the meaning of negation – we will deal with in Lecture 6. [Doubts about this in the context of an appeal to the logic of natural language will be articulated in Lecture 7.]

• Brady's treatment of the paradoxes of naive set theory and semantics relies crucially on the absence of TND. Brady blocks the step from

$$r \in r \leftrightarrow r \notin r$$

to the explicit contradiction

$$r \in r \wedge r \notin r$$

as this step requires TND (or equivalent principles). As we have seen, Routley takes this step by TND.