Lectures on Universal Logic

Lecture 10 – Adaptive Logics as Universal Logics

The Adaptive Logics program is a major group in current paraconsistent logic research. The main inspiration to develop adaptive logics (ALs) was to develop inconsistency adaptive logics. Meanwhile the research on adaptive logics involves logics for explanation and induction, partial revision, logics for ambiguities ...

General Idea

Modelling actual reasoning requires a dynamic modelling:

Depending on what information is *available at some time* inferences are drawn and may be *retracted*.

Common sense reasoning often is non-monotonic, i.e. $\Gamma \cup \{A\} \nvDash B$ where as $\Gamma \vDash B$

This non-monotonicity appears with respect to exceptions to some rule (default logic) or with respect to discovering inconsistencies in a given premise set Γ (inconsistency adaptive logic).

In case of the usual non-monotonic logics the dynamics in reasoning can be called an 'external' dynamics since it ensues by adding further premises/information to the premise set. External dynamics corresponds to the non-monotonicity of the inference (respectively consequence) relation.

In distinction to this adaptive logics are concerned with so called 'internal' dynamics: Within the reasoning on a conclusively given premise set the insight on logical relations within the premise set and its consequences increases.

Internal dynamics are concerned with the reasoning process itself.

Internal dynamic need not be mirrored in the (final) consequence set of a premise set Γ.

Extensions and retractions may cancel out in the end.

Modelling internal dynamics is closer to modelling real reasoning in discovering the power of a premise set. To do this we have to refer to the *stages* of a proof, and what has been established *so far* (without this 'so far' being dependent on new information extending the premises available at that stage).

Modelling internal dynamics is an extension of the Natural Deduction approach.

Retracting in the process of reasoning from a premise set cannot be completely avoided, since there is no general algorithmic procedure (for any logic) to test whether Γ or $\Gamma \cup \{A\}$ is consistent.

So we often extend our premise set Γ by a new assumption on the supposition that this extension is consistent, although it sometimes turns out not to be.

Especially if $\Gamma \vdash A$ *depends* on $\Gamma \nvdash C$ and no negative test (say in FOL) is available for $\Gamma \nvdash C$, then we have even no positive test for $\Gamma \vdash A$.

[The derivability of A may depend on the absence of C if say C states some exceptional condition on employing some rule to derive A.]

Retraction is of most interest with respect to internal dynamics, since given one and the same premise set the sentence A may be derivable at some stage and retracted later. A then might not belong to the final consequence set, but it appeared to during some stages of the reasoning process.

Lower and Upper Limit Logic

An adaptive logic is characterized by:

- an Upper Limit Logic (ULL)
- a Lower Limit Logic (LLL)
- an adaptive strategy.

The *Upper Limit Logic* (ULL) allows for the unrestricted application of logical rules to derive the most consequences possible. Here we have full classical re-capture.

Typically ULL is (standard) FOL. Since the set of consequences of this logic is larger, it is the *upper* limit logic.

The *Lower Limit Logic* (LLL) is chosen to model some type of restricted reasoning. In our case it is a paraconsistent logic, i.e. a logic that blocks the application of some rules of standard logic. Since the set of consequences is correspondingly smaller, it is the *lower* limit logic.

$$Con_{LLL}(\Gamma) \subseteq Con_{ULL}(\Gamma),$$

The *adaptive strategy* is the way to handle the management of restrictions and the corresponding retractions.

An adaptive logic such defined with a paraconsistent logic (PL) as LLL could behave like a flip-flop: It behaves like ULL if the premise set Δ is consistent and behaves like the LLL (PL) if Δ is inconsistent. But such a behaviour (and such a logic) is not interesting, since it simply means to revert to LLL in case of inconsistency. Adaptive logics do not do that in general: An

AL generates a set of consequences of an inconsistent Δ that can lay between $Con_{LLL}(\Delta)$ and $Con_{ULL}(\Delta)$. That is, the interesting case is:

$$Con_{LLL}(\Delta) \subset Con_{AL}(\Delta) \subset Con_{ULL}(\Delta).$$

Some rules (e.g. Disjunctive Syllogism [$\neg A, A \vee B \models B$] and *ex contradictione quodlibet* [$\neg A, A \models B$]) are not valid in many paraconsistent logics. Given their non-validity one cannot employ them to derive consequences.

But this is too restrictive: These rules fail only if A is a true contradiction. They could be applied to consistently behaving sentences. The idea of adaptation is therefore: Think of these rules as applicable and make exceptions *only* if one of the premises is known to be inconsistent. Since we do not know beforehand which premises *are* consistent, we may employ these rules incorrectly. That is why there is retraction. Retraction occurs (in the case of a paraconsistent LLL) if some sentence to which one of the restricted rules was applied turns out to be inconsistent. The application of that rule is retracted then. All consequences of that application are retracted as well.

Examples:

Some rules like Disjunctive Syllogism [$\neg A, A \vee B \vdash B$] and *ex contradictione qoudlibet* [$\neg A, A \vdash B$], and all derived rules depending on them, have to be restricted.

Restriction means here that they are only used if the on the left-hand side of the application no contradiction is involved.

1. Disjunctive Syllogism leads to triviality if A in $\neg A, A \vee B \vdash B$

   is a contradiction.

2. *ex contradictione quodlibet* leads to triviality if A in:

   $\neg A, A \vdash B$ is a contradiction.

3. *Modus Ponens* (in some PLs like **LP**) leads to triviality if A in

   $A, A \supset B \vdash B$ is a contradiction.

4. *Modus Tollens* (in some PLs like **LP**) leads to triviality if B in

   $A \supset B, \neg B \vdash (\neg A)$ is a contradiction.

Without adaptivity we had to reason using some PL in all contexts which we suppose to contain contradictions. Given that quite a lot of standard logic is missing [including contraposition, transitivity (of identity) etc.] that is a severe restriction. We cannot capture a lot of (harmless) consequences in that field then.

Philosophy as that area of universal talk about semantics and epistemology would have to use such a restricted logic. It is questionable how many of its theses and arguments could really (i.e. without hidden recourse to standard logic) be expressed.

Adaptivity, on the other hand, makes clear that reasoning from the present contradictions is rather the exception than the rule. Concerning some inconsistent theory T one can say, “We keep T – except for the pernicious consequences of its inconsistency”.

Types of Adaptive Approaches

Most ALs are *corrective*, i.e. LLL $\subseteq$ ULL = FOL.

An AL needs an *adaptive strategy* to deal both with inconsistencies (exceptions) *and* the problem that an inconsistent premise set need not tell us which premise is to blame (for example take $\{\neg p, \neg q, p \vee q\}$).

The most common strategies are the *Reliability* strategy and the *Minimal Abnormality* strategy. Depending on the chosen strategy the consequence set of an AL is greater or smaller.

Dab(Δ)-Formulas

Given a premise set Γ one likes to know which of them may be abnormalities. *Abnormalities* here are formulas of the form $A \wedge \neg A$ (we also say in this case that A is an abnormality, a contradiction).

Some premise sets might be such that we know:

(1) $(p \wedge \neg p) \vee (q \wedge \neg q)$

whereas neither disjunct is a consequence (so far). So maybe each of them or either “p” or “q” behaves abnormally.

“Dab(Δ)” abbreviates the disjunction of $(A \wedge \neg A)$ for all $A \in \Delta$. “Dab(Δ)” then expresses that at least one of the premises in Δ is abnormal.

“Dab” means “disjunction of abnormalities”. We are looking for *minimal* Dab-formulas (since the less disjuncts a Dab-formula has the more premises we have excluded as suspects).

Besides the formulas occuring in a Dab-formula there might be formulas which are *already* known as being abnormal. In general:

$$\Gamma \vdash_{LLL} (A \vee Dab(\Delta)) \text{ iff } \Gamma \vdash_{ULL} A$$

Here Δ contains the formulas on the consistency of which the application of some rules used in deriving A depends. If Δ is empty even LLL allows to derive A. If Δ is not empty LLL only ascertains the inconsistency (the disjunction of inconsistencies) and does not derive anything (like A) from it.

Note:

1. Δ typically is taken as the set of formulas in Γ that are or may be unreliable.
2. Hopefully $\Delta \neq \Gamma$, but, of course, $\Delta \subseteq \Gamma$.
3. Even if some formulas have been established as abnormal, there may be further formulas that are suspected as possibly abnormal; i.e. there may be a set of inconsistencies $\Lambda \subseteq \Gamma$ with $\Lambda \cap \Delta = \varnothing$. Dab($\Delta$) then expresses that there is at least one more member of Λ but we do not know which disjunct it is.
4. Formulas A for which $A \wedge \neg A$ has been proven are *unreliable per se*. Which A contained in Δ for which we have Dab(Δ) are taken as *unreliable* depends on the adaptive strategy.

Adaptive Strategies

One can follow the *Reliability* strategy that considers all $A \in \Delta$ as unreliable, or the *Minimal Abnormality* strategy, which with respect to (1), for example, assumes that once we consider the one abnormal we can take the other as normal (i.e., we can derive more consequences, since less exceptions are now operative).

A logic following the Minimal Abnormality strategy does not coincide in its consequence set with a logic that employs the reliabilist strategy.

In cases like (1) one can get indeterministic proofs following the Minimal Abnormality strategy: One chooses either “p” or “q” as being abnormal. Depending on the choice some proof steps using the non-chosen suspect formula are accepted.

If at some later stage in a proof one can *derive* one of the disjuncts in (1) [in general in: Dab(Δ)], then (1) [or Dab(Δ)] is no longer *minimal*. So, this Dab-formula (which – remember – is a disjunction) is replaced by one stating that derived inconsistency. Retractions based on the supposed inconsistency of one of the other disjuncts are taken back then [by *marking/unmarking* lines in the proof, see below].

Proof Theory

Proofs look like Natural Deduction Proofs with a further column:

n.<k,...>   A   Rule, m, l   {B}

We number the lines and include in “< >“ the premises a line depends on, then follows the formula, then a column naming the rule applied to get this line and the lines used in that application. The 5th column contains the set of formulas (possibly empty) on the consistency of which the derivability of the formula depends. These sets are called ‘conditions’.
Conditions obey the following abstract rules:

> (RU) If $A_1 \ldots A_n \vdash_{LLL} B$, then from $A_1 \ldots A_n$ on the conditions $\Delta_1 \ldots \Delta_n$ derive B on the condition $\Delta_1 \cup \ldots \cup \Delta_n$.

The rule (RU) concerns rules of Natural Deduction which do not require in LLL the consistency of the ingredient formulas. B just inherits the conditional dependencies.
Rules requiring such consistency operate on

> (RC) If $A_1 \ldots A_n \vdash_{LLL} (B \vee Dab(\Delta_m)$, then from $A_1 \ldots A_n$ on the conditions $\Delta_1 \ldots \Delta_n$ derive B on the condition $\Delta_m \cup \Delta_1 \cup \ldots \cup \Delta_n$

In this case consistency assumptions for the formulas in Δ_m are added.
The last line of a proof is the stage that the proof has arrived at.
Now, if one of the formulas in the condition gets to be known as non-fulfilling the essential criterion (here: consistency) the line is *marked*.

This is the reliabilist *Marking Rule*:

> If $Dab(\Delta)$ has been established, take as unreliable all $A \in \Delta$, and given the set U_s of formulas that turned out to be unreliable at stage s of the proof: If for $A \in \Delta_i$, $A \in U_s$, then line *i* is marked.

So all lines that depend on the normality of any formula in the suspect set Δ are marked.

The marking rule of the Minimal Abnormality strategy is more complex, but says roughly:

> If for $A \in \Delta_i$, A occurs in some Dab-formula, then line *i* is *not* marked because of that Dab-formula *if* there is another disjunct of that Dab-formula which is taken as unreliable.

So, marking applies only if some known abnormal formula has been used to derive some line. Lines that depend on a marked line have inherited the condition by either (RU) or (RC) and are, therefore, marked as well.
Depending on the strategy – or the premise set – a line can get *unmarked* later, even in case of the reliabilist strategy, as soon as the blame can be placed on another suspect formula.
Proof Example:
Instead of simply writing "(RU)", "(RC)" the detailed rules are given.

1.<1>	$\neg p \wedge r$	PREM	$\emptyset$	
2.<2>	$q \supset p$	PREM	$\emptyset$	
3.<3>	$s \vee \neg r$	PREM	$\emptyset$	
4.<4>	$r \supset p$	PREM	$\emptyset$	
5.<5>	$p \vee \neg r$	PREM	$\emptyset$	
6.<1>	$\neg p$	$\wedge$E, 1	$\emptyset$	(RU)
7.<1>	r	$\wedge$E, 1	$\emptyset$	(RU)
8.<1,2>	$\neg q$	Contrap., 6, 2	{p}	(RC), marked at 10
9.<1,3>	s	$\vee$E, 3, 7	{r}	(RC), marked at 10 unmarked at 11
10.<1,5>	$(\neg p \wedge p) \vee (\neg r \wedge r)$	Dilemma, $\wedge$I, 5, 6, 7	$\emptyset$	(RU)
11.<1,4>	$\neg p \wedge p$	$\supset$E, 4, 7	{r}	(RC)

In line 10 we get to know that at least one of "r" and "p" is inconsistent, so lines depending on them get marked.
Given a Minimal Abnormality strategy *or* seeing in line 11 that "p" is inconsistent we can blame "p" for line 10 and unmark the lines depending on the consistency of "r". The Dab-formula in 10 is no longer *minimal* after 11 (so a reliabilist would agree with blaming "p").
Let us look at derivability:

...

8.<1,2>	$\neg q$	Contrap., 6, 2	{p}	(RC) marked at 10
9.<1,3>	s	$\vee$E, 3, 7	{r}	(RC) marked at 10 unmarked at 11
10.<1,5>	$(\neg p \wedge p) \vee (\neg r \wedge r)$	Dilemma, $\wedge$I, 5, 6, 7	$\varnothing$	(RU)
11.<1,4>	$\neg p \wedge p$	$\supset$E, 4, 7	{r}	(RC)

At stage 9 "s" is derived, but no longer so at stage 10 because of the marking. At stage 11 "s" is once again derived. "¬ q" is not derived from stage 10 on.

Given the dynamic character of the proofs one has to distinguish:

- derivability at some stage
- final derivability.

The latter can be defined:

> A is *finally derived* at line *i* of a proof at a stage *s* iff line *i* is unmarked at *s*, and whenever line *i* is marked in an extension of the proof, then there is a further extension in which line *i* is not marked.

Of course, this property is (in most cases) not recursive (nor r.e.). One would like to have criteria to make an assessment.

Even if final derivability is not recursive (i.e. we have to *guess* whether some formula derivable now is finally derivable) this resembles our actual reasoning where we (mostly) lack similar assurance against revision.

Note also: There is *nothing* dynamic about final derivability.

LP as LLL

Being concerned with universality, and since adaptive logics are said to use paraconsistent logics as LLLs one can use quantified LP (LPQ) as a paradigmatic paraconsistent logic with an intuitive semantics, and enrichable by a detachable conditional connective besides [⊃], as LLL and FOL as ULL. [The example above used LP (Priest's 'Logic of Paradox') as LLL.]

Using LP as LLL combined with an adaptive strategy that relies on minimalizing the number of abnormalities leads to the effect that inconsistencies once present are spread (i.e. further inconsistencies can be derived). That is because some rules of spreading, which are blocked in LP, are no longer generally blocked when LP is used as LLL in an AL [see the example below].

Nonetheless this spreading of inconsistencies does *not* lead to triviality. The important meta-theorem is:

$$(\exists A)\ \Gamma \nvdash_{LP} A \quad \rightarrow \quad (\exists A)\ \Gamma \nvdash_{ALP} A$$

That is: Given that LP can avoid triviality, so can the AL that is based on LP as LLL (here called "ALP").

The following example shows that:

$$Con_{LP}(\Gamma) \subset Con_{ALP}(\Gamma) \subset Con_{PC}(\Gamma)$$

with $\Gamma=\{p \wedge \neg p, (q \wedge \neg q) \vee r\}$ [i.e., $Con_{PC}(\Gamma)$ is universal because Γ is inconsistent!]. We follow again the conventions given above:

1.<1>	$p \wedge \neg p$	PREM $\varnothing$		
2.<2>	$(q \wedge \neg q) \vee r$	PREM $\varnothing$		
3.< >	$\neg(q \wedge \neg q)$	LP-Theorem $\varnothing$	RU	
4.<2>	r	$\vee$E, 2, 3	$\{q \wedge \neg q\}$	RC
5.<1,2>	$r \wedge \neg p$	$\wedge$E, $\wedge$I, 1, 4	$\{q \wedge \neg q\}$	RU
6.<1,2>	$r \wedge p$	$\wedge$E, $\wedge$I, 1, 4	$\{q \wedge \neg q\}$	RU
7.<1,2>	$\neg(r \wedge p)$	DeMorgan, 5	$\{q \wedge \neg q\}$	RU
8.<1,2>	$\neg(r \wedge p) \wedge (r \wedge p)$	$\wedge$E, 6, 7	$\{q \wedge \neg q\}$	RU

The contradiction is line 8 is *not* derivable in LP, since ($\vee$E), which is Disjunctive Syllogism, is not valid in LP. "$q \wedge \neg q$" is not taken as abnormal here, since there is no need to do so: "$(p \wedge \neg p) \vee (q \wedge \neg q)$" is no *minimal* Dab-formula given premise (1). So, we derive with line 8 something between LP and PC, relativising it on the normality of "q".

CLuN

CLuN is the LLL prominently used by Dederik Batens et. al. CLuN is a very weak PL. It is defined by the *positive* fragment of standard PC [all rules/axioms not containing “¬“]:

(A1) $p \supset (q \supset p)$

(A2) $(p \supset (q \supset r)) \supset ((p \supset q) \supset (p \supset r))$

(A3) $p \wedge q \supset p$

(A4) $p \wedge q \supset q$

(A5) $p \supset (q \supset p \wedge q)$

(A6) $p \supset p \vee q$

(A7) $q \supset p \vee q$

(A8) $(p \supset r) \supset ((q \supset r) \supset ((p \vee q) \supset r))$

(A9) $(p \equiv q) \supset (p \supset q)$

(A10) $(p \equiv q) \supset (q \supset p)$

(A11) $(p \supset q) \supset ((q \supset p) \supset (p \equiv q))$

added to positive PC is the axiom

(A12) $p \vee \neg p$

Rules: (MP) and *Uniform Substitution*

Since the negation in CLuN is very poor (by the absence of most negation rules/theorems) inconsistencies are isolated from each other. The above example of spreading inconsistencies does not go through in CLuN, since DeMorgan and LNC are not valid in CLuN. Notice that (MP) for “⊃“ is valid in CLuN, in contrast to LP.

Negation in CLuN is peculiar since $v(\neg A)$ does *not* depend on the subformulas in A [see “¬“ semantics]! It complies only with a minimal condition on paraconsistent negation: Paraconsistency does not require $v(A) = 1 \rightarrow v(\neg A) = 0$, but only $v(A) = 0 \rightarrow v(\neg A) = 1$ in case that gaps are to be avoided. This is also true in CLuN.

Replacement of proven equivalents is not valid in CLuN, nor is unrestricted replacement of identicals in quantificational CLuN with identity:

$$\vdash_{CLuN}[((\neg P(a) \wedge a=b) \supset \neg P(b)) \vee (\neg P(a) \wedge P(a))]$$

That is: Substitution of identicals fails for inconsistent objects.

[The other rules/axioms for quantifiers and “=“ are standard.]

Let us look at a CLuN-Example:

$\Gamma=\{(p \wedge q) \wedge t, \neg p \vee r, \neg q \vee s, \neg p \vee \neg q, t \supset \neg p\}$

1.<1>	$(p \wedge q) \wedge t$	PREM	$\emptyset$		
2.<2>	$\neg p \vee r$	PREM	$\emptyset$		
3.<3>	$\neg q \vee s$	PREM	$\emptyset$		
4.<4>	$\neg p \vee \neg q$	PREM	$\emptyset$		
5.<5>	$t \supset \neg p$	PREM	$\emptyset$		
6.<1,2>	r	$\wedge$E, $\vee$E, 1, 2	$\{p\}$	(RC)	marked at stage 8
7.<1,3>	s	$\wedge$E, $\vee$E, 1, 3	$\{q\}$	(RC)	marked at stage 8 unmarked at stage 9
8.<1,4>	$(p \wedge \neg p) \vee (q \wedge \neg q)$	$\wedge$E, Dilemma, 1, 4	$\emptyset$	(RU)	
9.<1,5>	$p \wedge \neg p$	$\wedge$E, $\supset$E, 1, 5	$\emptyset$	(RU)	

At stage 8 lines (6) and (7) are marked using the Reliabilist strategy, since $U_8(\Gamma)=\{p,q\}$, but at stage 9 we have $U_9(\Gamma)=\{p\}$. Line (7) becomes unmarked, since "$(p \wedge \neg p) \vee (q \wedge \neg q)$" is then no longer a *minimal* Dab-formula. So "s" is derivable, but "r" is not.

AN

Paraconsistent logics typically give up Disjunctive Syllogism to avoid Explosion. An alternative would be to give up Addition ($\vee$I: $A \vdash A \vee B$), which much more looks like the entrance point of arbitrary sentences B.

AN in distinction to CLuN validates all negation reducing rules and Disjunctive Syllogism, but does *not* validate Addition and Irrelevance ($A \vdash B \supset A$) in general.

AN is said to be closer to actual reasoning.

AN semantics is 3-valued; 0 is the only non-designated value.

Decisive is the third value "X" ("contradictory"), where a disjunction with a contradictory disjunct is false only! At the same time a conjunction with a contradictory conjunct is true only! Because of the former Disjunctive Syllogism is valid (if A and $\neg$A were contradictory and B false, $A \vee B$ wouldn't be true). Addition has exceptions: If $v(A)=X$ (i.e. designated) combining with $v(B) = 0$ gives $v(A \vee B) = 0$.

Let us look at an example including Addition:

1.<1> p	PREM ∅			
2.<2> $p \supset q$	PREM ∅			
3.<3> $r \supset s$	PREM ∅			
4.<1> $p \vee r$	∨I, 1	{p}	RC	marked at stage (7)
5.<1,3> $q \vee s$	Dilemma, 2, 3, 4	{p}	RU	marked at stage (7)
6.<6> $\neg p$	PREM	∅		
7.<1,6> $p \wedge \neg p$	∧I, 1, 6	∅	RU	
8.<1,2> q	⊃E, 1, 2	∅	RU	
9.<1,2> $q \vee s$	∨I, 8	{q}	RC	

Using Addition in line (4) requires “p” to be consistent. Seeing that “p” is inconsistent lines (4) and (5) get marked. The content of (5), however, is derivable in (9) given the consistency of “q”. (⊃E) being a variant of Disjunctive Syllogism holds unrestrictedly, i.e. is a (RU)-rule not requiring the consistency of the antecedent.

Semantics

Semantics for ALs can be considered in terms of the models of the LLL and the ULL involved.

Since these logics are (usually) given beforehand, *their* semantics is given beforehand, and the semantics of adaptive logic “only” concerns the relationship between their models.

$Con_{AL}(\Gamma)=Con_{ULL}(\Gamma)$ if Γ has ULL-models.

Γ has ULL-models if ULL is FOL just in case Γ is consistent. In general:

$$Con_{LLL}(\Gamma) \subseteq Con_{AL}(\Gamma) \subseteq Con_{ULL}(\Gamma)$$

In case of inconsistency the AL-models are *just those* LLL-models that behave standardly on the consistent part of the premises.

Adaptive logics are sometimes said to deny inconsistency “in the world” (i.e. dialetheism) and only handle reasoning “in the face of” inconsistency; but to make the premises true (in the adaptive logic, i.e. in the LLL), there are *true* contradictions as far as the reasoning is concerned: The semantics of an according adaptive logic (i.e. its LLL) has to allow for assigning *true* both to a sentence and its negation.

In as much as some LLLs are introduced with their applicability within an AL in mind we will need some semantics for them (e.g., the deviant semantics of CLuN where the evaluation function of a model v_M can inherit *non-compositional* assignments from a model independent evaluation function *v).*

The crucial deviant part of CLuN semantics looks like this:

- the set S of sentences is mapped by *v* to {0,1}, v_M agrees with *v* here.
- every object in the domain has a (pseudo-)name.
- the set of wffs of *the form* ¬A is independently (i.e., notwithstanding their composition) mapped to {0,1} by *v*.
- $v_M(\neg A)=1$ iff $v_M(A)=0$ or $v(\neg A)=1$ (i.e. v_M can inherit evaluations of negations from *v*).

Because of the last clause there are CLuN models that make "¬ (p ∧ q)" true, but make "¬p" and "¬q" false only!

Meta-Logic

The meta-theory is phrased in a standard logic/language, i.e. not in an adaptive fashion itself. Since a *theorem* does not depend on any premise (so no inconsistency can be exploited) the *set of theorems* of an adaptive logic comprises all the A such that $\varnothing\vdash_{ULL}A$. That is: The AL-theorems are just the ULL-theorems (as LP-theorems are just the PC-theorems).

Adaptive logics so to speak have no theorems "of their own". Looking at the *consequence relation* only one sees the selective character of applying certain rules.

Adaptive logics are adequate with respect to final derivability.

A general meta-theorem about derivability:

> $\Gamma\vdash_{AL}A$ iff there is a finite set of abnormalities Δ such that
>
> $\Gamma\vdash_{LLL}(A \vee Dab(\Delta))$ and $\Delta \cap U(\Gamma)= \varnothing$.

[Remember: U(Γ) being the set of disjuncts of the minimal Dab-consequences of Γ in LLL, *plus* all the proven single contradictions. So the condition says that the normality conditions are minimal excluding already proven abnormalities.]

Assessment

Adaptive logics take the ideas of dynamics and reasoning with FOL as default ('classic re-capture') seriously. That is their greatest merit.

The AL program contains several approaches closely linked to investigating proof dynamics.

The logics like CLuN and its variants seem to be – in their semantics – even more deviant than the Relevant Logics we looked at or some paraconsistent logics like LP.
An option we pursue in the next Lecture, therefore, is a combination of adaptive ideas with some more natural rules for negation and other connectives/quantifiers for some adaptive universal logic.