LECTURES ON UNIVERSAL LOGIC

Lecture 1 – Why Universal Logic?

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Where does the Question of Universality Arise?

- The question of a Universal Logic can arise once we have paraconsistent logics that can deal with inconsistent contexts. Might a logic which can even deal with inconsistencies not be a logic that can be applied everywhere, i.e. universally?
- Given the multitude of logical systems investigated and proposed in the philosophical/logical literature could there be (i) a common core of these logics or (ii) a best system among them?
- Given the role of logic in reasoning should there not be the one explication of this very system, i.e. our universal logic?

Universality in a Paraconsistent Context

- There is no simple summary to all of paraconsistent logic, since there are so many paraconsistent logical systems, divergent approaches and applications. There is, however, an interesting question that may arise at least for a dialetheist (i.e. someone who believes that there are true contradictions): Are we to be logical relativists, choosing a different system for each different purpose, or shouldn't we – in the spirit of semantic and epistemic closure (including the meta-logic within one's logic), so to speak – look out for a *universal system* that is able to deal with all these purposes.

- Note
  - (i) the relation to genuine philosophical issues (like universality in semantics and in one's treatment of epistemic concepts like 'knowledge')
  - (ii) the formal issue of identifying logic and meta-logic (an option available only for paraconsistent logics).

Weak Universal Logic

- A universal logic might be universal as a paraconsistent logic, i.e. in all fields in which we need *a paraconsistent* logic this logic can be employed and gives acceptable results. This may be called *the weak universalist program*.
- One may take the weak universalist program as being extremely cautious: One takes one's favoured paraconsistent logic – and sticks to it in *all* contexts. Since this paraconsistent logic can deal with contradictory contexts it can deal with any context, so it really is universally applicable.

- The problem with this extreme caution is that one loses all otherwise available consequences in consistent contexts (as FOL inferences may not be valid paraconsistently).
- Therefore, one may rather try to distinguish the type of context one is reasoning in. In praxis this could mean that we employ standard First Order Logic for all non- semantic or non-antinomic contexts and switch to paraconsistency only in our formalization of complete semantics (or, maybe, set theory).

Strong Universal Logic

- Or a truly universal logic can be employed *everywhere*, supposedly containing a way to distinguish consistent from inconsistent contexts, without loss of proper logical power in comparison to FOL ('classical recapture' in classical contexts). This may be called *the strong universalist program* .
- In case philosophy contains inconsistent and consistent contexts, and uses arguments valid only in consistent contexts it seems to need to follow a strong universalist program.

- Both the approach of Logics of Formal Inconsistency (LFI) and Adaptive Logics follow the idea to be able to distinguish contexts of a stronger logic (usually FOL) and contexts for a paraconsistent logic *within the system used*.
- The way they do it is completely different, however. In the LFI-approach the distinction what kind of context we have has to be given *beforehand*; only given the corresponding knowledge can we choose the appropriate formalization (i.e. use °A [i.e. A is consistent] or not). In Adaptive Logics we mark the supposition that some formula has to be consistent, a supposition that may be *revised* in the process of reasoning; no prior knowledge about the consistency behaviour of a context is required.

Approaches and Issues

- We will look at some approaches, their background theories, strengths and – possible – shortcomings.
- All this will be related to the philosophical issues around universality.

Routley’s Universal Logic

- Richard Routley wrote a manifesto with the nice title "Ultralogic as Universal". He defines: "A universal logic, in the intended sense, is one which is applicable in every situation whether realised or possible or not." So, this idea is tied to question of modality and inconsistent ontology – and Routley's *noneism*.
- Standard logic should be recaptured in those areas where it is valid, according to Routley; he keeps *Tertium Non Datur*, but claims that universal logic should allow for truth value gaps.

- He proposes the Relevant Logic **DKQ** the first-degree structures (i.e. without nested entailments) of which are those of the Relevant logic **RQ** (i.e. a quantified version of system **R**). [**R** contains all **PC**-theorems, for "⊃". *Modus Ponens* is only given for "⟶".]
- The semantics of **RQ** needs something like the Routley Star and a ternary accessibility relation; **DKQ** has, according to Routley, no finite characteristic truth tables for its sentential part. [R being an undecidable propositional logic.]

Brady’s Universal Logic

- Brady's Logic **DJ^{d}Q** is announced as a universal logic. Brady explicitly says that the aim should be to have a universal logic and not chose one's logic for each department of research. Standard reasoning should be recaptured in consistent contexts, again.

- In **DJ^{d}Q** neither Disjunctive Syllogism, nor Absorption, nor the *Modus Ponens* -Theorem (for "$\rightarrow$") nor *Tertium Non Datur* are valid! So Brady introduces "classical sentences" (p', q' ...) for which Disjunctive Syllogism and *Tertium Non Datur* should hold.

- Brady uses his system not just to avoid trivialization from antinomies, but to avoid the antinomies in the first place by the invalidity of *TND*! (The Liar and its negation are *both* taken as false, against the spirit of Convention (T) and what the Liar says, it seems.)
- In this – two sorted – formal system the power of standard logic is indeed recaptured, but in the process of *formalization* we have to know already which sentences (or predicates) *are* "classical", i.e. consistent!

Adaptive Universal Natural Deduction

- One may take one's favourite LFI-system or adaptive logic system as a universal logic. We will not pick some particular of these systems here, since both approaches have virtues and shortcomings in comparison to versions of the logic **LP**.
- So, let us try a little mixing of ideas! Let us develop here an universal logic as a system of natural deduction with adaptive features and extended with operators of modalities and inconsistency, with some relative of **LP** as the lower limit logic.

Against Logical Pluralism

- Logical Pluralism claims that there is not one logic but a plurality of equally good ways to formalize logical consequence.
- This contradicts the idea of a strong universal logic which is seen as *the* proper philosophical logic or *the* logic of our reasoning.
- Logical Pluralism faces severe problems.

The Common Core Problem

• Logical Pluralism is put forth as a claim that is true. All those who are not immediately convinced of the claim by merely understanding it have to be convinced by arguments. These arguments have to be valid, correct and convincing in some sense. What sense is that? It supposedly has to be a sense of convincing valid argument that can be directed at any audience whatsoever. Thus, it seems to use the common core of accepted argumentative standards, whatever other logical preferences the different audiences may have.

- Thus, it seems to be the intersection of different (applied) systems of reasoning. That would be some kind of *common core* logic.
- So, in presenting a general argument for pluralism pluralists seem to presuppose the very universal core of logic the existence of which they reject.

The Formal Common Core Problem

- The theory language of logical pluralism used as a meta-language when talking about the different ways to spell out being logical has a logical form. At the most comprehensive level (the level which takes *all* structural elements into account) there is one complex logical form. Given the possession of logical form, some particles/words may be singled out as 'logical vocabulary'. These logical words have their respective syntactic behaviour and meaning constitutive rules (truth conditions). Thus, the theory language of logical pluralism provides us with *a logic*, namely the logic that goes with its logical vocabulary.

• Once the meta-theory of logical pluralism is spelled out formally, the proponents of logical pluralism have spelled out a formalism ready to deal with any logic whatsoever, i.e. a universal framework.

The General Logical Form Problem

- Linguists of the transformational camp (and some others as well) claim that we have a highly specified innately fixed module for language acquisition, which comes with principles the parameters of which are the only elements left to be settled by regional languages. With respect to our ordinary talking and thinking there is no unsettled part of our grammatical assessment of sentences. Why should logic have come apart from language?
- This is even more questionable since language employs a 'level' or 'phase' of logical form in processing mental representations and at the interfaces to other mental modules. This level or phase of

logical form (LF) is highly constrained by both internal constraints of syntax (like *Government*) and external constraints of semantics (like providing the structure for employing the quantificational truth conditions).

- There is overwhelming empirical evidence for this level of structured descriptions.
- Thus, Logical Pluralism seems to stand in conflict with a well-established tradition of treating logical form in linguistics and cognitive science. Logical Pluralism seems to be empirically wrong.