A Modal Tableau Method

The usual tableau rules for First Order Logic can be extended to the modal systems by introducing a set of modal tableau rules for "☐" and "◊".
Which conditions govern the relation of *accessibility* depends on the modal system considered.

1. To test any consequence relation $\Delta \models \Sigma$ between a (possibly empty) set of premisses Δ and a set of consequences Σ, draw a tableau with two columns, one labeled "T"(true) and one labeled "F"(false). Write the premisses in the T-column and the consequences in the F-column.
2. Label each of the columns at the origin of the tableau "1". Each time a tableau has to be split to deal with alternatives or a new tableau has to be introduced label the alternatives or the new introduced tableaus with numerals "2", "3" etc. Alternatives introduced *within* a tableau n are *sub-tableaus* of n. Rows positioned above the point of splitting the tableau into sub-tableaus are parts of both alternative sub-tableaus. (Note: A newly introduced tableau n+1 is *not* a sub-tableau of tableau n.)
3. If a new tableau m+1 is introduced by the rules for the modal operators occuring in tableau m, this tableau m+1 is *accessible* from m. If sub-tableaus are introduced into m corresponding subtableaus have to be introduced in m+1 and these are accessible from the sub-tableaus of m.
4. If ☐α occurs in column T of a tableau n, add α to the column T of the tableau *and* add α to the column T of all tableaus which are accessible from tableau n.
5. If ☐α occurs in column F of a tableau n, add and label a tableau n+1 which is accessible from tableau n and has α in its F column.
6. If ◊α occurs in the column T of a tableau n, add and label a tableau n+1 which is accessible from n and which has α in its T column.
7. If ◊α occurs in the column F of a tableau n, add α to the column F of all tableaus which are accessible from n.
8. The tableau at the origin and the tableaus introduced form a *tableau landscape*.
9. A sub-tableau is *closed* iff for any formula α, α and $\neg\alpha$ occur in one of its columns. A tableau is closed iff it either has sub-tableaus and all its sub-tableaus are closed or for some formula α, α and $\neg\alpha$ occur in one of its columns. If an accessible (sub-)tableau closes the accessing (sub-)tableau closes, and vice versa. A tableau landscape is closed iff all its tableaus are closed.
10. Apply in each (sub-)tableau the modal and the usual First Order Rules (with the usual restrictions on the quantifier rules), unless the tableau is closed or no further rule can be applied or no rule can be applied without repeating a sequence of steps already occuring in the (sub-)tableau.
11. A consequence is *valid* iff its tableau landscape is closed.