

# The Inexpressibility of Gaps in Ordinary Language

Many 3-valued approaches to the antinomies can be rejected as insufficient, because they either face strengthened versions of the antinomies or because they employ distinctions in semantics which they cannot express themselves. Such gap theories fall prey to Strengthened Liars and ‘Revenge’, which reintroduce the bifurcation needed for the antinomic reasoning (cf. the introduction and the papers in the volume *Revenge of the Liar* [Beall 2007]).

Pragmatic approaches to the antinomies often suffer from their proximity to such 3-valued approaches.

In this paper I explore the idea of gaps in evaluations again.<sup>1</sup> The crucial idea to be tested rests in the assumption of a (meta-)semantic rule which can enforce the retraction of a supposed evaluation of an antinomic sentence or statement.

We start, however, with a discussion of an attempt to deny the existence of token-reflexive truth value bearers in ordinary language.

## §1 Token-Reflexive Sentences and Statements in Ordinary Language

The word ‘this’ can be used as a deictic expression roughly synonymous with ‘that’ or as a strictly reflexive indexical. So

- (1) This sentence has five words.

can mean – in one reading to be disambiguated –

- (2) That sentence (over there) has five words.

or it can mean – in another reading to be disambiguated –

- (3) This (very) sentence (I am using right now) has five words.

In expositions of the antinomies one often uses ‘this’ in the sense of (3), e.g.

- (4) This statement is not true.

As the ‘this’ is strictly reflexive its reference does *not* vary from occasion to occasion (of using sentence (4)).

Statements result when a speaker uses a sentence on an occasion. Sentences often are indexical and

---

<sup>1</sup> Note on usage: an ‘evaluation’ assigns a truth value, an ‘interpretation’ assigns meaning, so that there may be an interpretation without an evaluation.

thus are neither true nor false as they stand. The indexicals have to be anchored to a situation of usage. *What is said* on that occasion then can be true or false. The distinction has found use not only in pragmatics, but is part *inter alia* of the framework of situation semantics. Statements are eternal (cf. Strawson 1952). Statements can be presented by *eternal sentences* or by some abstract representation (like ‘infons’ in situation semantics or state of affairs in other ontological frameworks).<sup>2</sup>

In these abstract representations any indexical part present in the sentence used (say “I sit on that chair”) is replaced by the item/referent talked about in the statement: “I” is replaced by the speaker, “sit on” by the relation of sitting on, “that chair” by that chair, including times and places, and so forth.

Therefore: An occasion of using a Liar sentence gives rise to a Liar statement, which is eternal, and refers to this very statement, and the property of truth, and so forth. For the statement (i.e. this objective content, which could also be specified as a tuple of items including that very statement) it is of no importance how the items are referred to.

Thus when someone reasons about (4) that (4) has no semantic value, and thus is not true, he *is not making another statement* than (4) does (or somebody using (4) does). He can speak French or whistle, as long as some conventions of fixing reference are in place he is making the same statement. Of course his speech act takes place on another occasion than mine, two tokens of sentence (4) are used. But he makes the same statement that anybody makes who uses (4). And as his reasoning leads back to the statement made by (4), the antinomic reasoning proceeds as it always does. One may be misled by the usage of ‘this’ or other natural language renderings of the antinomies, but shortcomings of phrasing the antinomies in these ways should be surmounted by formalization.

Some of the problems concerning whether some sentences really ‘say’ something or not might be solved by trying to be more precise on the formal side of the sentence/statement distinction. Unless one believes that natural languages are beyond formal treatment there have to be some formal means/representations which capture that distinction. Logicians exploring the logic of assertion and rejection sometimes use the syntactic derivability sign (i.e.:  $\vdash$ ) or some new device (like:  $\dashv$ ) to capture assertive force. Assertive force can be represented in natural languages themselves, which, of course, pretty nicely explains that we can write papers about it.

Now, as the sign of assertive force might define what distinguishes a statement from other ways of using sentences, and as this operator/sign is part of the formal machinery of language, one can

---

2 As statements can at least be expressed by eternal sentences I follow the practice of speaking of (eternal) sentences as truth value bearers.

distinguish well formed formulas containing it at the proper place from others. Thus we can define the class of statements (or statement representing eternal sentences). As this is a syntactic identification, the process of doing so is primitive recursive. Therefore we have all reasons to believe that the antecedent conditions of the *Diagonal Lemma* are fulfilled, and thus *that there is some sentence  $\lambda$  equivalent to the sentence that asserting  $\lambda$  results in a false statement*. As no indexicals are involved that sentences  $\lambda$  not accidentally is usually taken as the eternal sentence representing the Liar (statement). Once we have that eternal sentence in the context of an assertion sign it seems to be pretty *ad hoc* to deny that it says what it says. Even if no one bothers to utter it right now someone may do so, and  $\lambda$  just has the objective content it eternally has.

There are, thus, token-reflexive sentences in ordinary language. Even if “this” in (1) usually refers to another sentence (has an unspecified argument position), we can substitute a name of the diagonal of that sentential function into that position thus arriving at a token-reflexive sentence. From a syntactic point of view ordinary language seems to contain token-reflexive sentences. Otherwise ordinary language wouldn't be as expressive as some artificial languages.

## §2 A Semantic Rule for Gaps in Ordinary Language

How are we to interpret (4) then? Given standard semantics we arrive at the antinomy. – Can we avoid interpreting it at all? Even if we cannot make sensible use of (4) [see below], can we suspend (4) from having an evaluation? Given mildly realistic assumptions in semantics, even if we personally do not care about the sentence it has an evaluation, thus ordinary language will be inconsistent, whether we care or not.

One might resort to some meta-rule on evaluations that revises all evaluations in case they land us in contradictions.

Firstly, such a rule will be beyond standard logic towards default or adaptive logics (cf. Batens 2000 on adaptive, Besnard 1989 on default logics).

Secondly, if we do not distinguish object-language from meta-language this rule and its negation may enter into reflexive constructions. In contrast to a sentence, however, if there was a corresponding reflexive rule generating contradictions we simply need not adopt it. A sentence saying of itself that it was not subject to the original rule would be without evaluation itself.

The meta-rule had to say something like

- (MR) Whenever a sentence  $A$  on a supposed interpretation and evaluation (i.e. a straight forward application of the semantic rules) leads to a contradictory evaluation,  $A$  has no evaluation. The supposed evaluation is rejected.

The rule enforces and says in effect that we will never accept contradictory evaluations. It relativises the coverage of our semantic rules, which are otherwise understood as applying to all sentences. It is a rule about semantic rules. Given (MR) sentence (4) in fact has no evaluation. Relatives of (4) which token-reflexively say of themselves that they have an evaluation and that they are false are not simply false (as this would be an evaluation), but lack an evaluation themselves. (MR) regulates the absence of interpretations in general, even involving sentences referring (indirectly) to (MR), like

(4') If this has an evaluation, it is false.

(4'') This has an evaluation and is false.

Such sentences are without evaluations themselves. “This has no evaluation” can be consistently interpreted as false. So it seems that (MR) allows for consistent self-regulation and does not face a Strengthened Liar, so far.

Do sentences like (4) have a use in ordinary language?

In indirect speech we can, of course, mention them as examples of problematic sentences. In a straight forward assertive utterance the felicity conditions of such an utterance cannot be met, given (MR), as we assert as true a sentence which cannot be evaluated. In a liberalized understanding of the felicity conditions we may even allow for such utterances inasmuch as they succeed in putting forth a sentence to be evaluated, but are followed then by an act of retracting the assertion. The speaker might be blamed for violating the rule of asserting only what one believes to be true, or be blamed for believing this sentence to be true (i.e. for incomplete reasoning with respect to the sentence or neglect of [MR]).

In any case, showing that we can achieve no purpose by using (4), or even showing that an attempted assertive utterance will misfire, does not by itself entail that (4) has no evaluation. We need the additional appeal to a meta-rule like (MR). In this respect the treatment of the antinomies in ordinary language explored here is *not* just pragmatic. It concerns our framework of semantic rules.

Further on, that (4) has no assertive use and receives no evaluation need not and should not entail that (4) has no interpretation, no meaning. We understand what (4) wants to say, because we employ our capacity to understand token-reflexive sentences. Many of them are harmlessly true, like “This is a token-reflexive sentence”.

We understand (4) inasmuch as we know which semantic rules are to be applied to it (compositionally). We derive an interpretation. Once we arrive at the supposed contradiction (MR) will be activated and no evaluation is delivered on (4). The real gaps are gaps with respect to the evaluation of sentences, they are not semantic gaps, say in the sense that some sentences have no

meaning.

This whole approach looks like a semantics with truth value gaps (or even interpretation gaps), and then a Strengthened Liar

(5) This is not true.

should land us in a new version of the old antinomy, because *being* not true by falling into the *gap category* seems to make (5) true, and we have regained an antinomy. Having *no* evaluation cannot be just another evaluation (included in the opposite of being true), then, without contradiction. Gaps must be *real gaps*. They have to lead to real gaps in evaluating discourse and to real gaps in drawing inferences. The semantically crucial issue is distinguishing the statement that (4) is not true, as it has no evaluation, from an evaluation. So far we have no theory how such 'real gaps' should work. Suppose, for the moment, that gaps do not fall prey to a Strengthened Liar. Nonetheless we have to be able to talk about them.

### §3 The Inexpressibility Argument

One way one should not try to circumvent a Strengthened Liar is by restricting “not” in (5) or related sentences to be some kind of 'inner' negation which carries us only from truth to falsity and from falsity to truth. This way we could not even express that sentences like (4), which have no evaluation, are therefore *not* contradictory (i.e. true and false at the same time). To express the theory of gaps and (MR) we cannot forsake a 'wide' negation (cf. Bremer 2008).

Meta-linguistic sentences talking of the predicate “( ) is true” also do not change the picture, as the old antinomies can easily be transferred into meta-linguistic versions like

(5') The predicate “( ) is true” cannot be successfully applied to this sentence.

which raise the same issues of contradictions or inexpressible semantic facts.<sup>3</sup>

We face problems of inexpressibility with our treatment of (5).

Saying of (5) that it has no evaluation should be true. Suppose it is. Then it *is* also true that (5) is not true (as being 'true' would be an evaluation). And it would be a limit of semantic expressibility if we could not say so. Suppose we could say so. Does *this* then *imply* that (5) is true after all? – No! No, because to derive this claim we have to apply some semantic rule for “( ) is true” to (5), the application of which will be retracted by (MR). How can we say then that (5) is not true? *Saying* that (5) is not true seems like an instance of the type of (5), at least we seem to make the same statement like (5), as discussed in §1. That saying of (5) that it is not true has meaning (an interpretation) seems to be not enough, since we seem to know that it *is* also true. Saying of (5) that it is not true should be evaluated as being true, but this brings us full circle to evaluating a statement

---

3 Left as an exercise to the reader.

having the same content as stating (5).

The only option seems to consist in a better understanding of what (5) really says given the presence of (MR) and real gaps in evaluations. Either (5) says the same as (4), namely says of itself that it has the semantic property of being false, and so we have dealt with it already, or we have

(5") This has no evaluation.

which is just false, since assuming it to be true leads to contradiction. So we may reason that either (5) has no evaluation (by being [4] in disguise) or (5) is false (by being [5"]). Saying of (5) that it has no evaluation is (5"), which is false. So if (5) *has* no evaluation we cannot say so truly. So barring inexpressibility we should say that (5) is false:

(6) (5) is false.

but if (6) is true, it entails

(7) (5) is not true.

which again has the same content as (5). Our sentence (7) about (5) not being true is, of course, not the token-reflexive (5). (7) differs in logical structure, *but not in content*.

And thus, given our reasoning about (5), (7) is false, and again we have not succeeded in saying truly of (5) that it is false. Thus we arrive at inexpressibility after all, whatever may be the semantic matter concerning (5). Of course this is disastrous, as sentences about gaps should not be without evaluation themselves. Especially the claims of the gap theory should be true *simpliciter*. Semantic properties should be expressible in ordinary language.

(7) does not *assign* an evaluation to (5), and thus the *desideratum* for a theory of real gaps mentioned at the end of §2 is *fulfilled*, but even abstaining from assigning an evaluation to (5) it expresses a semantic property of (5), which *prima facie* entails facts about evaluations.

What have we seen? – Even leaving Strengthened Liars to the side and exploiting the presumably universally applicable meta-semantic resources of ordinary language coupled with the resources of a logic dealing with defaults or retractions a gap approach violates the idea that ordinary language can express all semantic facts (crucially including those the gap theory wants to talk about).

## References

- Batens, Diderik (2000). "A Survey of Inconsistency-Adaptive Logics", in: Batens, D. et al. (Eds.) *Frontiers of Paraconsistent Logic*: Baldock (Research Study Press), pp. 49-73
- Beall, JC. (Ed.) (2007). *Revenge of the Liar*. Oxford (OUP).
- Besnard, Philippe (1989). *An Introduction to Default Logic*. Berlin et al. (Springer).
- Bremer, Manuel (2005). *An Introduction to Paraconsistent Logics*. Bern et al. (Lang Pub.)
- (2008). "Kearns's Illocutionary Logic and the Liar", *History and Philosophy of Logic*, 29, pp. 223-25.
- Strawson, Peter Frederick (1952). *An Introduction to Logical Theory*. London (Methuen).