Truth Value Talk Without Quotation

§1. Semantic Predicates, Semantic Operators

A side issue in the general philosophy of language and linguistics, but a central issue in formal semantics and meta-logic has been whether in case a language is able to express semantic properties (like truth) at all, these should be expressed by predicates or by operators. If a semantic property is expressed by a predicate the expression (typically an eternal sentence) having this property has to be quoted. If a semantic property is expressed by an operator which modifies a sentence to yield another sentence no quotation is needed. Unfortunately there are problems in the way of naively extending a language with semantic expressions. According to *Tarski's Theorem* a language containing the expressive resources of arithmetic cannot – on pains of inconsistency – also contain its own truth *predicate*. According to theorems due to Montague and Thomason even languages *below* the expressive power of arithmetic which use truth *operators* will be inconsistent, once the truth operator obeys some simple axioms characteristic of truth (value) talk. These theorems seem to exclude a language able to deal with its own semantics, a truly semantic universal language. Natural languages, however, seem to be able to deal with their own semantics. And they seem to yield some antinomies like the Liar by way of their expressive power.

The scenery changes once paraconsistent logics are used as tools in modelling semantics. In as much as antinomies/contradictions are allowed within paraconsistent semantics and logics one may reconsider the use of truth predicates and truth operators in one's formal language and modelling.

In relation to the other papers in this volume it might be helpful to be precise on the point to be established. The argument does not try to show that all quotation marks can be dropped in a language: talking about the syntactic properties of a word or sentence (like: "morpheme" is disyllabic) requires quotation devices (i.e. quotation marks or Gödel numbers). The argument does not even show that *all* semantic quotation marks can be dropped. The focus is on quotation in relation to assignments of "true", "false" and other semantic values of sentences/statements. Given a Davidsonian approach to meaning and reference an account of truth provides a theory of the core semantic concepts, so having a quotation free theory of truths goes a long way towards a quotation free semantics. Nevertheless there may still be a need for quotation is semantics, especially if sub-sentential units (like proper names or predicates) are described semantically. The point of the whole exercise here in the paper is to see clearer where we need quotation marks and where we do not, which kind of logic allows for dropping some quotation marks and which do not.[1]

§2. Dialetheism

Dialetheism is the claim that some contradictions are true. For anyone trained in standard logic and raised in the belief that already in antiquity Aristotle settled once and for all that there is the Law of Non-Contradiction dialetheism sounds not just false, but bizarre. On the other hand people contradict each other quite often and a couple of theories have turned out to be inconsistent. Nevertheless the people who held inconsistent beliefs have not (at the time of holding these beliefs) believed just anything, as the standard rule of *ex contradictione quodlibet* would have it. Thus paraconsistent logics (logics that invalidate *ex contradictione quodlibet* and thus can tolerate even provable contradictions) have gained interest and lots of them are investigated and explored nowadays (cf. Bremer 2005). Dialetheism is *strong paraconsistency* in the sense that one cannot just tolerate some contradictions, but one should endorse some of them. This certainly needs argument.

Ever since its arrival dialetheism has been met with the proverbial *incredulous stare*, not only because of the inconsistent ontology of Routley's *noneism* (Routley 1979), but also with respect to the dialetheist's claim that one can knowingly believe and assert contradictions. Priest in the paper introducing his "logic of paradox" **LP** (Priest 1979) admits that the thesis of dialetheism is a dialetheia itself. In his book *In Contradiction* (Priest 1987) he argues that one can avoid dialetheism being a dialetheia itself if one is prepared to give up contraposition for the conditional in *Convention (T).*

The problem about contraposition is the following. Given *Convention (T)*, say – for a start – with a truth predicate and some quotation device, we have for the Liar statement λ

(λ) λ is false.

($T\lambda$) $\text{True}\ulcorner\lambda\urcorner \equiv \lambda$

The dialetheist need not endorse

(TF) $\neg\text{True}\ulcorner\alpha\urcorner \equiv \text{False}\ulcorner\alpha\urcorner$

But the dialetheist endorses for some α:

(D) $\text{True}\ulcorner\alpha\urcorner \wedge \text{True}\ulcorner\neg\alpha\urcorner$

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[1] I have to thank an anonymous referee for requesting these necessary clarifications.

Now, for a (first naïve) dialetheist λ is true and false, thus, because λ is false, ¬λ is true, and false, because λ is true. Given that ¬λ is true, we have, by *Convention (T)*, that ¬λ. Now contraposition with respect to (Tλ) gives us

$$(T\lambda') \quad \neg\lambda \equiv \neg True\ulcorner\lambda\urcorner$$

But as the (first naïve) dialetheist endorses both λ and ¬λ we can detach both in (Tλ) and in (Tλ') and get

$$(\lambda') \quad True\ulcorner\lambda\urcorner \wedge \neg True\ulcorner\lambda\urcorner$$

Thus the statement of the dialetheist that some contradiction is true turns out to be a contradiction itself! That may be much more than the dialetheist bargained for.

Priest thus endorses giving up contraposition in *Convention (T).* Another option (taken up in so-called *Adaptive Logics*) is to restrict both *Modus Ponens* and *Modus Tollens* to detachment with consistent sentences (cf. Batens 1989, Priest 1991).

Nevertheless Priest defends that one can believe and assert contradictions. Up to now criticism of dialetheism has focused on the problems what the status of dialetheism itself is and how it may be possible to believe knowingly contradictions. It is argued here that within dialetheism the resources are available to claim that dialetheism is true only (i.e. not false at the same time). Furthermore there may be occasions on which it is rational to believe and/or even assert contradictions, without thereby positioning oneself on a slippery slope towards an attitude of "anything goes".

Paraconsistent logics can level the distinction between object and meta-language. A semantically closed language not only is able to talk about its own expressions, but does contain at the same time its semantic expressions. These semantic expressions need *not* be taken as predicates (like a truth predicate applying to the quotation of a sentence), but can be taken as operators instead. One arrives at a paraconsistent language/logic which allows truth value talk *without previously quoting* the sentences which are evaluated.

§3 Universality and Contradictions

The main motivation for dialetheism is *universality* as a feature of language and cognition. Universality means that we are aiming at – and supposedly capable of – a theory of, say, language *in general*, that is not just of this or that language or languages of this or that formal structure. And this theory is expressed in language, so that at least some language can be its own meta-language (with respect to all interesting properties of that language, semantics included). Universality means as well that we use fundamental concepts like *denotation* or *true* unrestrictedly.

A language *L* is *semantically closed* if and only if *L* is syntactically closed and able to talk about its own semantics. The meanings of the terms of L can be given within *L* then. If a language is semantically closed it can not only talk about its own expressions (by suitable names or quotation marks), but it can also apply semantic properties to these terms, and even to the terms that express semantic properties:

(1) Sentence number (1) is grammatical.

(2) This sentence contains six meaningful words.

(3) The third sentence displayed in this list is true.

Here some formula refers to itself by a description/name occurring in that formula. An infamous example is the Liar:

(λ) λ is false.

The Liar is a fixed point for the predicate "( ) is false" (or "not-true"), saying "I am false". Now consider (λ): If λ is true, then λ is false, because the general term in λ "( ) is false" should apply to the singular term "λ". If λ is false, then λ is true, because λ is just saying that it is false. So we get:

$$(4) \qquad \mathrm{True}\ulcorner\lambda\urcorner \equiv \mathrm{False}\ulcorner\lambda\urcorner$$

or given that we have a two-valued logic where "false" is just the opposite of "true":

$$(5) \qquad \mathrm{True}\ulcorner\lambda\urcorner \equiv \neg\mathrm{True}\ulcorner\lambda\urcorner$$

So λ is an *antinomy*. There are more harmless semantically self-referential sentences, like (2). The Liar is the basic case of bad semantic self-referentiality. It bears its badness on its sleeve. It is not hidden. Within a language that is semantically closed semantic self-reference may be hidden, however. Suppose today you only utter a single statement

(6) What the pope declares today is true.

and *as a contingent matter of fact* he only says that day:

(7) Everything N.N. says today is false.

Dialetheism claims that it cannot be prevented, since a natural language *is* a semantically closed language. Since, furthermore, the antinomy can be *proved*, it has to be *true*. So the dialetheist has to show three things:

1. The contradictions can be *proven* in a *sound* non-standard logic, if we use a semantically closed language.
2. We *have to* use a semantically closed language.
3. There is *no satisfactory alternative* to accepting the antinomies (i.e. the attempts to prevent them either fail or have consequences worse than dialetheism).

I will skip the arguments for these three claims here (for the details cf. Bremer 2005, or Priest 1987), and focus rather on the dialetheist's method of talking about truth and antinomies. Priest shows in all formal detail how a (paraconsistent) language can contain its own truth predicate (cf. Priest 2006a, pp.125-40). He also shows in more recent work (cf. Priest 1997, 1999) that a (paraconsistent) language can contain its own *denotation* function (as applicable, say, to proper names or descriptions). Thus, adding this to what is to come in the next paragraphs, even *sub-sentential* semantic quotation can be circumvented to that extent by using a paraconsistent logic!

§4 Truth Operators

One may think that since naive semantics contains *Convention (T)*

(T) $T\ulcorner\lambda\urcorner \equiv \lambda$

and negation can have the standard truth table in many paraconsistent logics, a contradiction like (λ) is said to be both true and false (non-true), and the negation of (λ) is said to be both true and false (non-true). With contraposition and *Convention (T)*

(T') $\neg\lambda \equiv \neg T\ulcorner\lambda\urcorner$

one arrives – as we have seen in §1 – at the embarrassing result that $T\ulcorner\lambda\urcorner$ is itself both true and false. One may take this as a *reductio ad absurdum* of the dialetheist position. Talk of truth and unique commitment have to be reconsidered, therefore.

Truth concerns what is the case whether we believe it or not. Belief concerns what we are willing to include in our inferring. What we believe we take into account in our reasoning (belief is cognitive). Generally, being provided with reasons for α is seen as the basis for believing α, given that the reasons for some γ incompatible with α are not stronger. On a gullible approach to (perceptual) belief one believes every α one has no reasons against.
The best backing for a belief α is a proof of α. Having reasons is superior to mere belief in the truth of α. Having no independent access to the (ultimate) truth of α going with reasons is the rational way, whatever the (ultimate) truth value of α is or turns out to be.
Typically it is taken to be rational to assent to [to affirm] what one believes. Assertion is to assent to or to affirm what one believes. If one has a belief α one also has the disposition to assert α. One does not need additional reasons to proceed from believing to asserting. On the other hand, asserting α is done by a speaker confronting an audience (assertion is pragmatic). Asserting α is done with a purpose in view of an audience, so that this purpose exceeds using α in one's processes of deliberation. As an (speech) act with some purpose asserting α has to meet the basic felicity conditions of successful action plans, like the purpose being not achieved anyhow *without* my action and this specific action being *fit* to the purpose. Asserting contradictions *seems* to fail both conditions.
Dialetheism as a thesis should be asserted as being *only/just true* (i.e. not being false at the same time). One should be able to say, *without* saying something false, that a true sentence/statement is true. One should be able *to express the semantic properties* of all sentences/statements (including the antinomies).
Dialetheism claims that some contradictions are true. So we have some sentence λ with λ, $\neg\lambda$, $T\ulcorner\lambda\urcorner$, $T\ulcorner\neg\lambda\urcorner$, $F\ulcorner\lambda\urcorner$, $F\ulcorner\neg\lambda\urcorner$ to start with. The reasons for this are that these contradictions are *provable* given some unassailable principles and structures in a semantically closed language. Now, these antinomies being *true* and being *justified* as true, by proving them, give all the reasons to *believe* that they are true and thus to believe them (themselves). So a dialetheist should *believe*

(8) The Liar is true.

thus (9) The Liar

thus (10) The Liar is false.

Giving up believing what one has proven seems to be a desperate and *ad hoc* manoeuvre. So a dialetheist has inconsistent beliefs. She reasons *using both* $T\ulcorner\lambda\urcorner$ and $F\ulcorner\lambda\urcorner$ if necessary.
Now, paraconsistent logics can level the distinction between object and meta-language. A

semantically closed language not only is able to talk about its own expressions, but does contain at the same time its semantic expressions. These semantic expressions need not be taken as predicates (like a truth predicate applying to the quotation of a sentence), but can be taken as *operators* instead. One arrives at a paraconsistent language/logic which allows truth value talk without previously quoting the sentences which are evaluated.

Let us consider six such operators: "$\Delta\alpha$" says that α is true only, "$\nabla\alpha$" that α is false only, "$°\alpha$" says that α is consistent (i.e. has only one truth value), "$\bullet\alpha$" says that α is contradictory. We can then say – and these being *just* true – that the Liar is true, false, not simply true, not consistent, and so on. "T" and "F" are *now* understood as *operators* applying to formulas/sentences not quoted. These operators are bivalent themselves and are defined according to the table ('0,1' as truth value means that α is an antinomy):

α	**$\neg\alpha$**	**$T\alpha$**	**$F\alpha$**	**$\Delta\alpha$**	**$\nabla\alpha$**	**$°\alpha$**	**$\bullet\alpha$**
0	**1**	**0**	**1**	**0**	**1**	**1**	**0**
1	**0**	**1**	**0**	**1**	**0**	**1**	**0**
0,1	**0,1**	**1**	**1**	**0**	**0**	**0**	**1**

Using these operators dialetheism can fulfil the traditional condition on any decent theory: that it claims to be *just true* (and not only as true as its negation). We see firstly: Dialetheism is thus *no form of trivialism* (that everything is true). The trivialist proposes $(\forall\alpha)(T\alpha \wedge T\neg\alpha)$ or $(\forall\alpha)(T\alpha \wedge F\alpha)$. The dialetheist claims $(\exists\alpha)(T\alpha \wedge F\alpha)$, but also $(\exists\alpha)(T\alpha \wedge \neg T\neg\alpha)$, and $(\exists\alpha)\nabla\alpha$. And given some formal system some formulas can be exhibited having these properties (e.g., defining a *bottom particle* $\bot$ with $\nabla\bot$ being valid). $\top$ can be defined as the *top particle* with $T(\alpha \vee \neg\alpha)$, being true only. The bottom particle $\bot$ can be defined as $\nabla(\alpha \vee \neg\alpha)$, being false only.[2]

To have and use the (T)-scheme at the same time as these operators we need some revisions in the logic of the conditional, like giving up on the unrestricted validity of *Contraposition*. The language contains formula (e.g. $T\alpha \wedge \nabla\alpha$) that can be evaluated only as being simply false.

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2 Note that, in contrast to even the intuitionist negation rules, $\bot \equiv (\alpha \wedge \neg\alpha)$ need not hold if α is a dialetheia, since then $T(\alpha \wedge\neg\alpha)$, and ∇ is incompatible with T.

These formulas, of course, cannot be derived.[3]

To return to the semantic use of the truth operators: Saying $T\lambda$ is thus simply true: $\Delta T\lambda$. This does not exclude that $F\lambda$ is also simply true: $\Delta F\lambda$.

Now it *seems* that saying of the Liar that the Liar is false is just what the Liar is saying:

(11) $F\lambda \equiv \lambda$

Then we might have: (12) $FF\lambda$

and this contradicts $\Delta F\lambda$! But to derive (12) we use either

(13) $F\lambda = \lambda$ or

(14) $\vdash (F\lambda \equiv \lambda)$

The equivalence thesis (14) may be wrong. And substitution of identicals is one of those inferences *restricted to consistent objects* (to which λ does not belong). Even if (14) is not wrong deriving $FF\lambda$ supposedly has to use some form of detachment, which again is restricted to consistent sentences (to which λ does not belong).

Let us take it, thus, that $F\lambda$ can be believed and – being bivalent – can be asserted. Asserting $T\lambda$ or $F\lambda$ certainly fulfils some purpose, be it in explaining dialetheism or in arguing with opponents of dialetheism.

Asserting α [$A\alpha$] means claiming that α is true. One may thus have:

(15) $A\alpha \equiv AT\alpha$

Denying α [$D\alpha$] cannot be understood by a dialetheist as affirming $\nabla\alpha$ [$A\nabla\alpha$]. Denying α would thus be incompatible with affirming α (i.e. affirming $T\alpha$), as we supposedly have:

(16) $A(\alpha \wedge \phi) \equiv A\alpha \wedge A\phi$

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3 We do not need the details of all these restrictions here. The reader has only to know the general idea of paraconsistent logics and the idea of "adaptive logics" (Batens 1989, 2000) to restrict some rules to consistent sentences (respectively to retract some supposed consequences if the rules to derive them employed, against the restrictions, some inconsistent sentences). A paraconsistent logic like Priest's **LP** can be developed into an adaptive logic with a restricted form of *Modus Ponens* and *Modus Tollens* (Priest 1991). Within paraconsistent logics "logics of formal inconsistency" (Marcos 2005) employ consistency operators in the object language. Truth operators can then be added. Blending these approaches one can have an adaptive paraconsistent logic which combines the extensional and intuitive truth conditions of **LP** with the use of truth and consistency operators and restrictions on substituting identicals to consistent objects (cf. Bremer 2005, pp.224-37). Substituting identicals has to be restricted to avoid so-called *hyper-contradictions* (cf. Bremer 2005, pp.185-97). As adaptive logics restrict detachment to consistent sentences (a special sort of consistent object) the adaptive use of substituting identicals restricts substituting to objects which have not contradictory properties. As a consequence of these restrictions *hyper-contradictions* no longer go through, as Strengthened Liar reasoning in these languages depends on either detaching or substituting with the antinomic sentences or their names.

Thus

$$(17) \quad AT\alpha \wedge A\nabla\alpha \equiv A(T\alpha \wedge \nabla\alpha)$$

which is even too much for a dialetheist, as $T\alpha$ and $\nabla\alpha$ are incompatible.

One needs a distinction then between affirming $\nabla\alpha$ and affirming $F\alpha$ [$T\neg\alpha$]. Let us take affirming $F\alpha$ as *denial* and affirming $\nabla\alpha$ as *rejection* of α [$R\alpha$].

A dialetheist may then *both assert and deny* a sentence α:

$$(18) \quad A\alpha \wedge D\alpha$$

As this is equivalent to

$$(19) \quad AT\alpha \wedge AF\alpha \qquad \text{[i.e. } A(T\alpha \wedge F\alpha)\text{]}$$

Whereas there are situations in which a dialetheist accepts both α and $\neg\alpha$, or $T\alpha$ and $F\alpha$, there are no situations in which a dialetheist *accepts and rejects* α at the same time. As the foregoing distinction shows there is, furthermore, one kind of contradiction that (even) a dialetheist cannot support:

$$(20) \quad \neg(A\alpha \wedge R\alpha) \qquad \text{[i.e. } \neg(AT\alpha \wedge A\nabla\alpha)\text{, i.e. } \neg A(T\alpha \wedge \nabla\alpha)\text{]}$$

since $T\alpha$ and $\nabla\alpha$ are semantically incompatible.

Another simple point is that no-one (including the dialetheist) can have *pragmatic* contradictions: Speech acts being bodily movements that either occur or do not, there is no pragmatic parallel to having it both ways, i.e.

$$(21) \quad \neg(A\alpha \wedge \neg A\alpha)$$

This instance of the accepted tautology $\neg(\alpha \wedge \neg\alpha)$ expresses not only a semantic exclusion the dialetheist accepts (and sometimes nevertheless supersedes), but the absence of the mysterious feat of asserting something and not doing it at the same time.[4]

Thus without sliding into mystery or being silenced one can be a dialetheist and claim some crucial antinomies to be true. Dialetheism itself is not a paradoxical statement, but the theory that fits the aspirations of a universally minded philosophy.

A crucial ingredient is to drop the object-/meta-language distinction and use (bivalent) semantic operators.

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[4] One may also consider introducing *Rejection* as a speech act *sui generis* (i.e. as not being defined by *Assertion* and truth operators). One may then have a – arguably – more proper pragmatic theory of the different attitudes and the distinctions between asserting the opposite, withholding one's opinion and rejection. An outline of these ideas can be found in: Bremer 2008, pp.79-90).

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