On the very idea of revision of logic

§1 When scientists get into trouble with their theories it is theories not reality which are revised. If your biological account of an organ, say the kidney, provides no coherent explanation of the data you cannot revise the kidney, your account of its structure and function has to adapt. The same applies to the organ brain ('mind/brain' as is sometimes said).

Seen from this perspective the very phrase "revision of logic" has a misleading tone to it. Compare the case of languages: You can chose to talk German if you are able to when doing business in Germany; you can chose to speak Esperanto to impress your peers; but you cannot chose to have no natural language at all. Despite differences in approach and detail linguists agree that humans possess a language faculty, which is uniform species wide. The mind is not a blank slate. The language faculty has an initial state containing principles and parameters to be set. From this perspective (you may call it the 'cognitive science perspective' or the 'Chomskyan perspective') the same applies to logic. Humans possess – besides or as a part of – the language faculty a logic faculty or module that comes with a certain structure of principles. This structure is as it is, there is no room for 'logical pluralism' here. Theories of logic share the fate of linguistic theories: they have to be revised if incoherent in face of the data. Theories of logic are revised, logic isn't.

§2 The questions to be raised now are: 'Are theories of logic in any way different from ordinary theories in cognitive science?' and 'What are the dimensions on which theories of logic should be evaluated?'

A crucial difference might be that logic is considered to be normative. Following proper logical rules helps to infer true (or in any other way designated) statements from the other true (or in any other way designated) statements. Some rules may be more appropriate in some contexts than other rules. Thus we come to see some formal system ('a logic') to be used on some occasion and not on an other. Logic thus seems up to choice. Call this the 'logical positivist' or 'Carnapian' perspective on logic.

Choosing logic cannot be regarded as the whole truth for the simple, but fundamental, reason that in choosing a logic the mind cannot be a blank slate. Some core principles have to be operational in deciding on an applied logic. This core may be the logic faculty. Further on, normativity does not stand in conflict with explanatory theories. Compare linguistics again: Norms do not cease to be norms just because you describe their structure and give a (coherent) account of their function and what following them achieves.

Thus there is room for Carnap's 'Principle of Tolerance' in choosing applied/regional logics, but behind and besides this we can study the core logic of the logic faculty.

§3 After these preliminaries we have to turn to the dimensions on which theories of logic are evaluated, and what reasons may be given to prefer some set of rules to another set (which in terms of 'revision' can be read as: what reasons can be given to revise standard FOL).

In analogy to the general philosophy of science we have to look at the issues:

(1) What are the data a theory of logic has to account for? (To be considered are the issues of 'intuition', 'access', 'psychological reality' and reflective equilibrium.)

(2) What are the criteria of better coherence in case of a theory of logic?

Paradoxes can be considered as a *heuristic* to assess the coherence of a theory of logic, respectively its accompanying set of rules/axioms. A paradox/antinomy shows that a set of rules/axioms is not maximally coherent, has limited application.

In addition to meeting some standard criteria of coherence a theory of logic has to meet further criteria as being part of a comprehensive theory of cognition, like

(3) feasibility (of the set of rules in complexity measures)

(4) being embeddable into a wide (partially evolutionary/naturalistic) theory of cognition (which raises, for instance, the issue of evolution going for working solutions in standard environments, not for principled solutions).

Lastly a theory of logic has to

(5) relate logic to epistemology and logic's function with respect to achieving epistemic virtues.

§4 What are the data for a theory of logic? On the one hand we can observe how people reason. Collecting examples and generalizing – maybe by disregarding supposedly obvious errors – one may thus come to a corpus of somewhat idealized ordinary argument patterns. (Generalizing and idealization are not completely unproblematic here, but no more than in other areas of science.) On the other hand a theory of meaning (for logical vocabulary of just for words in general) will come with a set of inferences based on meaning, thus being logical. Bringing these two sources together Nelson Goodman in *Fact, Fiction and Forecast,* John Rawls in *A Theory of Justice* and others have developed the idea of (wide) *reflective equilibrium*. The equilibrium has to take our intuitions of validity into account. Given a re-construction of the inferential rules and meanings (of logical vocabulary) involved, some of these pre-reflective intuitions can be superseded. Paradoxes (like the 'paradoxes of material implication') and antinomies (provable contradictions) play the role of abnormalities and recalcitrant data. A theory that can explain them away or accept them scores higher on the observational requirement of meeting the data than theories which do not.

Preferably capturing the rules of logic in some area of reasoning aspires to the following two ideals:

I. *Intuitive Correctness*: The inferences underwritten by the logical systems are intuitively valid.

II. *Intuitive Completeness*: All the inferences considered to be valid intuitively can be derived using that logical system.

Within cognitive science the ideal of reflective equilibrium has be extended to the idea of *wide reflective equilibrium*: One has to consider not just our intuitive judgements of validity, but also constraints of cognitive (computational) complexity and learnability (in a social or evolutionary context).

Another important constraint concerning the data basis of logical theory is

III. *Accessibility:* All inference principles of the logical systems have to be cognitively penetrable.

In contrast to syntactic principles in linguistics, which are often or mostly processed sub-doxastically, rules of inference have to be accessible to rational agents and speakers to some degree (have to be 'cognitively penetrable'). Rules of inference are employed and appealed to in communication and deliberation. Justifying assertions involves in principle the appeal to inferential procedures and standards of argumentation. These cannot be completely beyond the ken of the agents/speakers participating. Thus a logical theory postulating inaccessible principles can be ruled out. A logical theory containing gerrymandered or highly complex principles we cannot understand on first hearing is at least put in doubt.

§5 General philosophy of science adds to the *observational requirement* of a theory fitting the data, preferably all the data (captured in the requirement of 'data completeness') criteria for a coherent structure of a theory. We consider here: *simplicity*, *explanatory power*, *consistency*.

§5.1 *Simplicity* comes as ontological simplicity and as methodological or structural simplicity, which is equivalent to explanatory power.

Ontological simplicity may concern either the number of types of entities allowed for in a theory or the number of entities (of some/any kind) allowed for in a theory. In the case of logical theories a contentious posit are possible worlds. Possible worlds have become common parlance in semantic model theory. One lesson to be learned here may be: As competing logical theories all employ possible worlds they are in the same boat with respect to that measure of coherence; criteria of coherence (and theory choice) can be indecisive in face of our best theories if they share the features related to these criteria of coherence; but if one theory stands out from the crowd of its competitors in that feature the scales can be moved in its favour.

Back to possible worlds: Suppose – unfortunately contrary to common practice – that the theories which employ possible worlds are clear about what they mean possible worlds to be. If “possible world” is only a title for some set theoretic structure we have only a case of wrong advertisement. The interesting case comes with the assumption of possible worlds as entities *sui generis*. In that case there seems to be an argument involving ontological simplicity available: Most theories that employ possible worlds already employ abstract entities (like sets). Models are set theoretic structures. If models are around anyway then models can stand in for possible worlds. Ontological simplicity decides in favour of models and thus against possible worlds. The argument can only be toppled by an appeal to explanatory power (i.e. that possible worlds are needed to explain semantic or logical features unexplainable otherwise). Typically (with the exception of David Lewis in *The Plurality of Worlds*) such arguments are missing.

Simplicity in the number of entities seems to be unimportant as most logical theories allow for an infinity of entities anyway. This need not be so, however, if some version of finitism can be sustained. The supply of expressions of a logical system need not be endless, but may be indefinitely large, so that in all practical employments of the system we never run out of expressions. If there are not infinitely many numbers (or what not else) then the logical meta-theory can employ finite set theory instead of standard set theory (like ZFC). Apart from dealing with finite collections only finite set theory has also the explanatory advantage of containing absolute complements and a universal set.

§5.2 *Simplicity in explanation* (mostly considered as '*explanatory power*') is the key criterion of theory choice. A theory with simpler principles has more explanatory power as less or simpler principles cover the same ground as more or more complex principles do in other theories (given, of course, both theories fulfil the observation requirement). In case of logical theories theories involving less principles/rules or reduction sets (of logical vocabulary) may thus be preferable to those more complex. An interesting debate around that issue may be Michael Dummett's case for intuitionism. Dummett claims in *The Logical Basis of Metaphysics* that the intuitionistic rules for logical junctors and quantifiers are more appropriate than the standard rules as the intuitionistic introduction rules (in natural deduction) match the elimination rules; he states his case for some 'harmony' between these rules (some 'Harmony' with capital 'H' some 'harmony' without) as they are independent of each other, thus the rules for negation conservatively extending conditional logic, and so on. Dummett tries to establish 'harmony' as a new criterion to prefer a logical theory – as simplicity disfavours his account: Propositional logic can be reduced to a single logical junctor (say the Sheffer stroke). That one junctor allows to derive the complete set of propositional junctors, thus covering the maximal ground. It is much simpler to assume that the logic faculty comes equipped with the Sheffer stroke than to assume a set of junctors each independent of each other. Dummett appeals to theories of learnability, but an appeal to evolutionary theory may outweigh that: We can easily imagine that evolution equipped a cognitive system with the capacity to recognize that two things/states can not be the case together. With this standard propositional logic was in place. Any further developments might proceed from there, but have to use that core as point of departure.

§5.3 *Consistency* was commonly – before the advent of paraconsistency – seen as a precondition for anything to count as a theory contender. A theory leading us into an antinomy is usually rejected. Even if paraconsistency (at least in the form of dialetheism) allows for some contradictions being true not just any contradiction in one's logical theory are acceptable.

We have to distinguish here between a logical theory being inconsistent and a formal system allowing for inconsistency. Consistency works as a constraint in paraconsistency as strong arguments are needed to overrule that requirement (i.e. the presence of the Law of Non-Contradiction) within a formal system. One type of argument put forth by the dialetheists refers to simplicity: Some of our fundamental logical or semantic principles (like the Truth Schema or Naïve Comprehension) lead to antinomies, but these contradictions are acceptable as true contradictions, since these principles thus keep their maximal generality and simplicity (not to mention the failures of competing theories in that area). Another type of argument by proponents of paraconsistency refers to the observational requirement: Most people will not infer from some contradiction to any statement whatsoever; *ex contradictione quodlibet* is not underwritten by most people's logical intuitions. This means that logical rules that incorporate the *quodlibet* (like Disjunctive Syllogism or Modus Ponens) have to be understood as restricted in some fashion. In terms of formal systems this might mean that Modus Ponens has to be taken as a non-universal rule (like in a Default Logic or some Adaptive Logic).

§6 In addition to the general criteria of coherence a theory of logic inasmuch as it concerns cognition has to meet the further requirement of *feasibility*. Information storage and processing in humans is constrained by the general capacities of human brains and the affordable resources of deliberation in situated action. Results of computational complexity may not be easily transferred to human cognition (as complexity measures, for instance, work with worst case measures in the long run, where in applied cases an exponentially complex computation may be feasible on the usual input or a polynomial complex computation may involves too high a polynomial degree to be feasible on even small input). Nonetheless result of computational complexity theory might provide a rough assessment which rules of inference are more feasible than others. In case of alethic modal logics of necessity a further case can be made for S5 on basis of such feasibility reasoning. Propositional logic is NP-complete (by SAT being NP-complete). A modal extension of propositional logic which does not increase complexity of computing validity is *prima facie* preferable to an extension which increases complexity of computation. As S5 allows for reduction of modalities S5 is also NP-complete. In contrast weaker modal logics like K or S4, both of which involve many more basic modalities, are in a different complexity class: PSPACE. Thus S5 is vastly more feasible. Some modal logics that contains simple relational reasoning move to complexity classes EXP and EXPSPACE. A logical theory involving rules of exponential complexity has at least to add a supplementary theory what (cognitive and computational) shortcuts may help to decrease this computational bottleneck (e.g. chunking statements or allowing for small error probabilities instead of certainty).

§7 Having a proper theory of logic, and modelling the human logic faculty thus follows roughly the same methodology that other (empirical) theories of cognition do. As there is the human language faculty, there is a human faculty of logic. As linguistic theories are revised to capture the initial state of the language faculty and its growth, also distinguishing competence and performance, so logical theories have to be revised in their attempt to capture in a formal system the core logic of the human logic faculty, and to account for a possibly gap between the strength of that system and its pragmatic employment in situated deliberation and communication. Evolution might revise logic, logicians revise logical theories.