Logic

Logic is at present an interdisciplinary and thriving area of study. Logic is related not just to mathematics and philosophy but also to formal linguistics, theoretical computer science and the cognitive sciences in general. Therefore we see on closer inspection within the field of logic a plenty of subfields, logical systems and areas of research. Research is directed at meta-logical investigation (concerned with formal systems abstractly conceived) as well as directed at many areas of applied logics (relating formal systems to their application in linguistics or using them to model technical systems and their behaviour). Within cognitive science one may further ask how much of logic is "psychologically real" (in the human mind/brain).

The central topic of logic is the validity of *consequence*. The class of those inferences and argumentations (whether in science or everyday communication) or those state transitions in technical systems is to be found in which a true conclusion is arrived at from (a possibly empty) set of premises. Logic is not concerned with the truth of the premises/assumptions, and thus not with the question whether these premise force the conclusion on us, but only with the connection between the premises and the conclusion. This connection should be strict in case of valid inferences. "strict" here means that the conclusion cannot be wrong in case the premises are true. Valid inferences have to preserve truth. As theory of valid inferences logic analyses the form of the premises and of the conclusion and uses a formalism. Logic is formal inasmuch as it is not a *collection* of valid inferences, but is concerned with those inferences which are valid because of their *form*. An inference is valid by its form if it can be formalized such that it can be brought in at least one valid form of inference (a form of valid consequence). Of course any argument can be brought in a form in which it is not valid. Only valid arguments, however, can be brought into a valid form. A form of inference is valid if in case of substituting non-logical words of the same syntactic category consistently for other words of the same syntactic category the form is still valid. So "If A then B, now A, thus B" is valid since substituting any sentences for A and B (consistently in both of their occurrences) produces another instance of this valid form, of *Modus Ponens*. Some expressions (like in the example the "if … then ---") are treated as logical vocabulary. Which expressions belong to the logical vocabulary (the logical constants) depends on how much logical structure should be revealed. Propositional logic is only interested in expressions like "if…then" or "and" that connect whole sentences/propositions. First Order Logic (looking at predicates and singular terms) investigates the inner structures of the sentences that keep unanalysed within propositional logic. In this way subject-predicate and quantificational structures like "for all x: if x is A, then x is B" are revealed. Depending on the depth of logical analysis corresponding levels of logical vocabulary and constituents are brought out in the formalism. Which level one looks at depends on the supposed context of the argument: If the argument is already valid at a propositional level one need not look deeper.

Logical truth is only of secondary interest. Logical truths can be defined as those conclusion that follow even from an empty set of premises. They are (always) true just because of their (own) form. Typically axioms are considered to be logically true and the transformation rules of a system have to preserve this logical truths to the derivable sentences (the theorems). In elementary logics (like propositional logic) there is a logical truths for every valid consequence because of the *Deduction Theorem*. There are, however, logics which contain valid consequences but contain no logical truths at all! Inheriting truth form premises to conclusion is something we are *always* interested in. Axiomatization may be a supplementary means to systematize a body of knowledge.

Validity thus is in modern logic a matter of (logical) form. Formal validity is a sufficient criterion for validity. Logical form not only shows that some inference is valid, but also shows why it is valid. In this way logicians develop formal systems and logic is the science and the study of *formal systems*. Formal systems may be used not only to prove the validity of some reasoning but also to model some knowledge system. Proposing a formal system for some concept (say *necessity*) may, further on, be part of a conceptual analysis or re-construction of the logic or meaning of "necessity". Logical explication of this kind is one of the interesting uses of logic for philosophy or theory of language.

Modern logic thus is symbolic logic. Traditional logic was concerned not only with inference but also with a theory of concepts and judgements. Many areas that nowadays are treated in philosophy of language or philosophy of science were part of traditional logic. Inferences were not as much systematic as in modern logic. Modern logic possesses a systematic theory of inference because inferences are considered as part of a formal system, a calculus. With respect to a calculus and its intended application (say a expressing quantificational structure of explicating the concept of modality) one can ask whether the formal system with its derivational structure is adequate with respect to the intended semantics. This question can be split in the questions of *correctness* and the question of *completeness*. Given that we have an intended area in view that the supposed logic is believed to formalize we can ask whether all the derivable sentences that the system delivers are valid in the intended sense. This is the question of correctness. For example, we may ask whether any quantificational theorem that First Order Logic delivers, is logically (i.e. always and under any interpretation of the non-logical words) true with respect to a domain of individuals sorted into sets according to their properties. And we may ask whether any logical truth, that holds because of the inherent structure of such domains of individuals, can be arrived at with the formal derivations of First Order Logic. This is the question of deductive completeness. When we learn that some calculus is correct we know that we cannot go astray in using this calculus. When we learn that some calculus is complete we even know what given enough cunning we may arrive at anything that there is to be logically known about the formalized field. So we can be sure – which we could be and were not in case of traditional logic – that we have captured the essential logical constants and structures and that we have a complete logical understanding of the formalized field. In this way the meta-logical proofs of *adequacy* that we have for many propositional logics and for standard First Order Logic, and many non-standard logics also, are of outmost importance. Many propositional logics have the further feature of being *decidable*. That means that we not only have a formal procedure to derive at theorems, but that we can also set up a formal procedure that after finitely many steps tells us whether some sentence is a theorem of the system or not. Unfortunately when infinity and quantificational relational structures combine decidability is typically lost. First Order Logic is therefore not decidable (only fragments which are either finite or not concerned with relations decidable). First Order Logic, however, is compact. *Compactness* means that if a conclusion follows from some set of premises this conclusion follows from a finite subset of these premises. This is an important property since it shows that we do not have to take in an infinite of premises to see whether something is a First Order logical truth. Stronger logical systems (like Second Order Logic) lose these properties of deductive completeness and compactness. They have, on the other hand, more expressive power, since they also quantify over properties, not just individuals. Because of their stronger expressive power axiomatic theories in Second Order Logic have the property of being categorical. *Categoricity* means that all models of these theories share a unique structure. For example one may look at the natural or real numbers. Arithmetic and analysis as formalized in Second Order Logic delineate a unique structure, whereas in First Order Logic we can also define arithmetical concepts, but cannot exclude that there are non-intended interpretations of these axiomatized theories (i.e. interpretations which do not deal with the natural numbers).

One cannot, therefore, say that there is just one best logic for mathematics or formal analysis. This finally becomes abundantly clear when we consider the variety of modal logics and the many applications of non-standard (e.g. many-valued) logics. These logics can sometimes (as mostly in case of modal logics) be seen as extension of standard-logic. We consider then not just individuals and their properties, but also, say, temporal structures. These logics have a wide application in modelling formal systems. Non-standard logics or "deviant" logics call into question some of the inference forms of standard logic. Relevant Logics, for example, question whether really anything follows from a set of inconsistent premises. Typically we would not infer anything bizarre, only because it turned out that our beliefs or our data bases were inconsistent.

What the manifold of formal systems shows is that an argument never shows something unconditionally. First of all it depends, of course, on its premises. But, secondly, it also depends on the logic it uses. And whether some logic is the appropriate choice in some context or application may be a non-trivial problem. This makes the current debates about logical pluralism versus logical monism/universalism so exciting and philosophically important.

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