

# Elucidating the Alethic Modalities. A Fictionalist Account

## §1 *A Reductive Analysis of Modality?*

Many accounts of the alethic modalities<sup>1</sup> like ‘possibility’ and ‘necessity’ try to be reductive. Modalities are supposedly explained by providing truth conditions for modal statements in a semantics that does not contain modalities itself, but refers to some *sui generis* entities like ‘propositions’ or ‘possible worlds. One may ask whether there could or should be any reductive account of the modalities at all. Modality may be an irreducible semantic concept, and all we can do is to elucidate it by some model (e.g. some type of ‘possible worlds’ talk). Modality may point to a fundamental feature of reality.

The reason for this scepticism concerning reduction rests in the hidden modal assumptions made with respect to the entities that are employed to explain modality. The very term “possible world” points to such presuppositions. These presuppositions may hide in some construction principle (like ‘independence’ of the building blocks of a combinatorial account of modality) or be given with assumptions of consistency. For example, ‘consistency’ explained as the non-derivability of a contradiction rests on ‘derivability’. ‘derivable’ is a (hidden) modal notion (as witnessed by the “-able” in the English term). One is not saying that the contradiction has been derived, but – sic! – that it could be derived, i.e. that it is *possible* to derive it.

There may be reductions of modality, however – *inter alia* versions of consistency accounts. Nonetheless even a non-reductive elucidation (e.g. in terms of consistency) may be illuminating. It may (a) establish meaningful modal talk; modal talk could be meta-semantic talk (with respect to consequence) being mirrored in the object-language (like in Provability Logics). It may (b) be part of a full-blown metaphysical picture (like Modal Realism).

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<sup>1</sup> In the following ‘modalities’ for short.

## §2 Linguistic Fictionalism (I)

Linguistic ‘ersatzism’ as an account of the modalities provides a version of a *fictionalist account of possibilities*: the possibilities do not exist (neither in the space-time universe nor anywhere else in reality), there are no possible existants, there are only *stories*. Or stories about them *might be written*. If the stories just ‘might’ be written the account cannot be reductive. It can be *reflective*, however: ersatzism *is* a story about possibilities itself. It tells how and why such stories might be written.

## §3 Explicating Modalities

Rudolf Carnap in *Meaning and Necessity* aims at an explication of modal terms in terms of his semantic construction of state *descriptions* and Meaning Postulates. He does not supply a formal system of modal logic, although his suggestions point to something close to S5. Modality is modelled by means of a theory of formal languages. The main idea is that all complete re-combinations of basic terms (singular terms and general terms) which *do not* contradict the Meaning Postulates (including logical axioms) constitute a state description, the logical closures of which are the possible worlds.

This means that  $\alpha$  is possible iff there is a possible world  $w$ ,  $w \models \alpha$ . This in turn (by the definition of “possible world”) means:  $\not\models \neg\alpha$  iff  $\models \diamond\alpha$ . This is not S5 inasmuch as S5 is deductively complete with respect to some (standard) possible worlds semantics for it, and for this  $\diamond\alpha$  should be *derivable* if valid (i.e. true with respect to all possible worlds). But to know whether  $\diamond\alpha$  is valid, one has to know  $\not\models \neg\alpha$ , i.e. a *negative* fact about derivability. This not just makes it dependent on a (hidden) modal concept like derivability, but is a fact which in the interesting cases is not decidable in general. It will be decidable in principle given finitistic restrictions on the number of basic singular terms (individuals) and general terms (properties).

That this conception is not deductively complete, however, does not make it useless.

Statements of the type  $\diamond\alpha$  are epistemically difficult to assess, but such epistemological difficulties do mean neither that we do not understand what the statement says nor that we do not know how to argue for such a statement’s truth or falsity. Modal statements need not be epistemically simple. They are not on any of the main accounts of modal semantics.

$\diamond\alpha$  is if true *not* a derivable truth in a sufficiently expressive First Order System, as for provable (“B”)  $\nVdash\alpha \Rightarrow \vdash\neg B\alpha$  is not valid in the logic of provability (by *Gödel’s Incompleteness Theorems*). So, although  $\nVdash\neg\alpha$  is a semantic/logical property of the system,  $\diamond\alpha$  cannot be a derivable truth in such systems in all cases of  $\alpha$ . So, strictly speaking, Carnap’s supposed system is *not* deductively incomplete, as  $\diamond\alpha$  is not a consequence that can be *expressed* in general as a logical truth *in* the system.  $\diamond\alpha$  is true by the logical/semantic rules of the system, thus a logical/semantic truth, but a truth *about* the system, not a logical/semantic truth *in* the system.

$\diamond\alpha$  could be derivable in a paraconsistent system in which Gödel sentences are (just) further antinomies – in a system in which the meta-reasoning about derivability is done in the system itself. Thus – given semantic closure combined with an application of the *Church Turing Thesis* (CTT) of capturing our (meta-)reasoning within a sufficiently extended (paraconsistent) formal system – one may argue:

- i. Suppose:  $\nVdash\neg\alpha$  is true given a system of inference.
- ii. Then:  $\nVdash\neg B\neg\alpha$  as the argument for (i) is existing *within* the system of inference.
- iii. Thus:  $\nVdash B\neg B\neg\alpha$  again as the argument for (ii) is existing *within* the system of inference.
- iv. Thus, by definition:  $\nVdash B\diamond\alpha$
- v. Thus:  $\nVdash\diamond\alpha$  by the plain correctness of “B” [ $\vdash B\alpha\supset\alpha$ ], as *Löb’s Theorem* [i.e.  $\vdash B(B\alpha\supset\alpha)\supset B\alpha$ ] does not apply in a paraconsistent context.

#### §4 *Modality as Meta-Semantic*

Alternatively to such an approach one could proceed in a fashion of elucidating modality without logical/semantic closure by starting from the observations just made:

- i. there being a (often hidden) dependency on derivability
- ii. possibility being the *meta*-property of ‘possibly true’ with respect to sentences of a formal system.

and see them as a way to *forsake* a philosophically loaded primitive notion of possibility altogether. One could claim that  $\diamond\alpha$  is just an object-language rendering of a meta-language statement, namely one of *satisfiability*:

$$\models \diamond\alpha \text{ iff } \exists M \models \alpha \text{ for some model } M \text{ (i.e. } \alpha \text{ being satisfiable)}$$

And the claims about satisfiability and the existence of models can – given the presumption of at least correctness if not completeness as well – be further traced back to statements about a story’s *consistency*:

$$\models \diamond\alpha \text{ iff } (\exists s)(\alpha \in s) \text{ and } s \not\vdash \perp$$

i.e.  $\alpha$  is part of a (complete) consistent story (a negation-complete consistent set of sentences). A consistent story  $s$  has a model, thus:  $\alpha \in s, s \not\vdash \perp \Rightarrow \exists M(\forall \gamma \in s) \models_M \gamma$ , i.e.  $\exists M \models_M \alpha$ , i.e.  $\models \diamond\alpha$ .

## §5 *Linguistic Fictionalism (II)*

Possibility is thus reduced to consistency, where consistency has a modal element in talking about the derivability of sentences.

A realist with respect to abstract entities who considers a formal system as an abstract object, which exists even without our successive epistemic access to it, can *eliminate* the *residual* model element in ‘derivable’ and simply state  $\not\vdash \perp$  as a *fact* given the system as it is. This (“ $\vdash$ ” meaning “there *exists* a derivation”) would be a complete reduction of modality.

Thus, on the one hand “possible” as a term can be reduced ultimately to a syntactic concept, which thus elucidates it in a regimented form. An ‘explication’ in Carnap’s sense is achieved. On the other hand, we see that the basic syntactic notions contain an aspect of modality *if* we restrict ourselves to talk in terms of our limited (epistemic and deductive) abilities.

## §6 *Metaphysical Modalities*

An account of this sort would, it seems, take all modalities to be *de dicto*; there are no modal properties ascribed to entities independently of linguistically established modalities. The fact that a formal system might be able to express *de re* modalities is in itself no reason to

consider the respective sentences (possibly) true. One might give their truth conditions in a way that leads back to *de dicto* modalities, e.g.  $\exists x \diamond F(x)$  may be seen as making a *de dicto* claim for all assignments to the variable (i.e. some sentence being true of that object).

But *de re* claims seem to make sense. In the object-language  $\diamond \alpha$  says not of a sentence but of a state of affairs that it is possible. Modal talk in the object-language applies to the world. We say what is possible or not *in* the world. Derivatively we ascribe modal properties to entities in the world. They have them themselves (in that sense *de re*).

These *de re* modalities, nevertheless, go back to the ways we in our theories and in the Meaning Postulates<sup>2</sup> of our language describe or conceive of the world. We have chosen these ways of talking and formulated our theories, on the other hand, *because* we want our language and theories *to fit to* reality. By our *de dicto* modalities we try to trace any inevitable (i.e. exception forbidding) objective connection in reality. The strength we attach to some connection determines whether we see it as semantic or just empirical.

Considering just semantic axioms we can talk of a broader class of possibilities than if we are taking the empirical assumptions of our best theories into account as well. We see links (between properties) of different strength and we want to capture the differences in strength. Consistency with respect to some empirical theory elucidates empirical possibility (i.e. compatibility with the laws of nature). Consistency with respect to semantic axioms elucidates logical/semantic possibility.

The contrast between metaphysical and linguistic possibility should not be understood in a way that any linguistically *based* elucidation of modality rejects the distinction between the strength of some connections between properties. Our best theories and corresponding Meaning Postulates try to trace the structure of reality. If they are true, these connections are *there*. ‘Metaphysical possibility’ is then not to be contrasted to ‘semantic possibility’: in our best theories they should coincide.

## §7 *Necessary Existents*

If modalities are elucidated by Meaning Postulates and the semantic and syntactic properties of a formal system, there are no exclusions with respect to sentence types that are considered

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<sup>2</sup> Meaning Postulates are just axioms including non-logical concepts without any presumption of providing conceptual analysis.

possibly or necessarily true. So, existential claims can be possibly true (if the concept of the entity involved contains no contradiction) – or even be necessarily true. If there are Meaning Postulates/Axioms making *existence claims* these existence claims are – *prima facie* in the *shallow* sense of a Carnapian explication of ‘analytic’ (as ‘following from the Postulates’) – derivable as theorems, thus being necessarily true. In the non-shallow sense – the sense to be elucidated – they are synthetic (as they do not decompose by the form of an implication the meaning of a term, as most Meaning Postulates). So, in that sense postulating them renders them *synthetic a priori*.

These necessary existence claims – prototypically in mathematics – may be part of our best theories, thus we understand that necessary existants are part of reality. Controversial posits are entities like the ‘perfect being’, necessarily existent.

Our linguistic frameworks are not directly proven themselves (as they set out what counts as a proof). In that sense synthetic *a priori* sentences in them cannot provide a – non-shallow (i.e. not just axiom repeating) – *proof* of a necessary existant as postulated. They can be used in proving other necessary existants conditional on the ones postulated. Our frameworks are viable in the holistic pragmatic fashion that ultimately serves as our best available justification of proceeding with these frameworks and believing their theorems and assumptions.

## §8 *Linguistic Fictionalism (III)*

A *linguistic ersatzism* avoids the postulation of (new) kinds of *sui generis* entities: necessarily existing abstract propositions which do not contain their subject matter as constituents (which these do not to avoid possible existants and overlap between such propositions which take on the role of possible worlds, as abstract stories)<sup>3</sup>. If one has other reasons besides an account of modality for this type of entity, they come in handy: As they need not be constructed (like real sentences) a reduction of modality seems possible. The possible is the realm of these complete, conjunctive propositions (standing in for possible worlds). Supposedly inconsistent propositions just do not exist. In contrast one has to explain why supposedly inconsistent sets of sentences are not constructed (or are not constructible). This – as an ontological investment – solves a problem of analysis by fecund ontological

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<sup>3</sup> An example is Alvin Plantinga’s *The Nature of Necessity*.

postulation. Nonetheless we see a postulation here which inherits all the epistemological and metaphysical problems of postulating abstract entities.

A linguistic ersatzism also has advantages over a non-linguistic account in the tradition of Ludwig Wittgenstein's *Tractatus logico-philosophicus*, which deals in a combinatorial account of the modalities using a non-abstract ontology, the major ingredient of which are states of affairs.<sup>4</sup> The problem such an account faces is to talk about possibilities ('possible states of affairs') without either taking them as sentences or as abstract entities (like propositions). There seems to be no place left to place such entities. A 'possible states of affairs' cannot *be* a recombination of the constituents of actual states of affairs (i) because these are parts of the actual states of affairs *already* (and at least the individuals cannot be replicated), (ii) because *if* they were combined thus, they *are* combined, i.e. one would have actual states of affairs.<sup>5</sup> So where are the combinations? One seems to land on a general principle:

(\*)  $\diamond\alpha$  iff the constituents of  $\alpha$  could be combined in that fashion.

This is, of course, no longer a reductive explanation of modality, but just a substitution of *possible combination* for *possible truth*. A reductive account might proceed on the general principle

(\*\*) All combinations of atomic individuals and atomic properties are possible.

This shows the alignment of such a theory to Logical Atomism. This explanation now rests on the assumption of independently existing atomic constituents. This might be an option, but it certainly faces the epistemological challenges (i) to identify such atomic constituents, and (ii) to analyse all complex individuals and properties in their terms. No one has delivered on these desiderata – presumably relegated to a completed science! The problem was – at least – involved in the downfall of Logical Atomism.

Linguistic ersatzism, therefore, remains the best option. So long as it relies on the idea of sets of sentences being 'constructible' and (semantic) consistent in adhering to previously given axioms or Meaning Postulates it cannot provide a reductive explanation of the modalities. It

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<sup>4</sup> An example is D.M. Armstrong's *A Combinatorial Theory of Possibility*. The ontology of non-transcendent universals and states of affairs Armstrong sets out in *A World of States of Affairs*.

<sup>5</sup> The problem resembles Bertrand Russell's problem in his attempt of a *Theory of Knowledge* with having individual negative states of affairs which make negated sentences true.

can, however, provide an elucidation of our modal talk and the role of modal talk in our linguistic frameworks. At the same time, it avoids more controversial ontological posits.

## §9 *Modal Instrumentalism*

A fictionalist account involves many intricacies and might be cumbersome to handle at least in its semantics expressed with *stories* (about entities) for modal talk and existing *entities* of various types for non-modal talk. Quantifying-in and counterfactual reasoning about existing objects raise then technical intricacies in formalising them within one (object) language. Higher order quantification aggravates the complexities.

The point of setting out the conception of fictionalism, however, is not to propose working with a formal system that mirrors fictionalism and its claims properly. The point is to have a theory that shows how modal analysis *could* be done without extravagant ontological commitments. This resembles a nominalist or fictionalist account of numbers – one sets out the account and then goes on to use the standard formal systems in the knowledge that their efficiency and seeming simplicity of expression is valuable, i.e. taking an *instrumentalist* stance on their ontology, especially set theory and model theory. The same can be done in modal logic: One may use higher order intensional/modal logic with a model theoretic framework (in the broad sense of including inaccessible cardinals or classes ...) including an ontology containing possible worlds and possible entities (of whatever type). The fictionalist just does not believe in these scaffolding structures (i.e. the ontological talk taken literally).

An argument for scientific realists to be modal instrumentalist may stress the difference between a commitment to unobservables of some kind and a commitment to non-existing entities (whatever “non-existing entity” means).

