

Frege's Theory of Sentential Unity and Russell's Theories of Propositions

§1

Frege provides a theory of sentential unity. The distinction between concept and object he takes to be of crucial importance and to be one of his quintessential insights. The category distinction between concepts and objects explains sentential unity and why no further constituents can be added to a sentence at will.

Russell provides us with no theory of sentential unity. In fact his early (*The Principles of Mathematics*) theory of propositions seems to be unable to explain why no further constituents can be added and why we cannot simply get a proposition out of a collection of objects ('terms' in his then vocabulary).¹ Russellian propositions taken as abstract entities also seem unable to distinguish active and passive renderings of a sentence, as well as a complete reversal in the way Θ -roles of a predicate are filled.

§2

Do we need a theory of sentential unity?

Suppose we take (with Frege and Russell) truth to be basic (i.e. we do not try to define the concept 'truth'). Then we can separate statements from other sentences or word collections as those linguistic items that can be true or false: as the basic semantic unit as far as claims concerning reality are made. They are then a natural collection. We need not further explain what distinguishes them, supposedly not even what *makes* them true or false or what this quality consists in.

When we now look inside propositions or sentences we may do so without the purpose of

1 Reflections on Russell will easily suffer from mixing up the different views Russell developed between *The Principles of Mathematics* (1902) and *Introduction to Mathematical Philosophy* (1919) and *The Philosophy of Logical Atomism* (1918). The focus in this paper is put on some common ground of Russell's theories of propositions and some systematic questions relating to them, so that the constraints of interpretative adequacy are weakened here.

explaining their unity. Their unity is explained, as far as its special status is concerned, by their ability to bear truth values. It can be taken as basic as soon as we start with taking truth as basic: there have to be units which behave the way statements behave. We recognize these units (i.e. statements) by their relationship to truth.

This *ratio cognoscendi* on the other hand may have a *ratio essendi* – what about statements makes them prone to be bearers of truth values? Frege's theory explains the unity of them by the complementary features of their *constituents*. On a linguistic level Russell (at the time of the *Philosophy of Logical Atomism*) seems to follow suit at least in parts: propositional functions are defined as functions that have propositions as *symbolic units* as values. Propositional functions understood as schemata are working thus on the lines of Fregean concepts. The basic semantic unit (namely assertoric sentences/statements) has to be elucidated by analysis, and this analysis points to the crucial categorical distinction between the concept/propositional function and its argument(s).²

§3

For a theory of sentential unity we need the claim that the constituent structure of a sentence needs two categorically distinct components: distinct in their syntax and their semantics. Frege's distinction between concepts and objects provides just that.

(SU) Theory of Sentential Unity

- i. The unity of sentences involving first-order general terms (referring to first-order concepts) and singular terms referring to objects stems from the general terms (and the concepts they refer to) being unsaturated, them being saturated at their argument positions by singular terms referring to objects, which are saturated (as are their referents).

Frege's error lies in the move from the proper claim that some unsaturated expression needs to refer to an unsaturated entity, and that some saturated expression needs to refer to a saturating entity, to

2 Russell claims correctly then that we should understand general functions by first clarifying propositional functions, and not the other way round, as seems to be the case in Frege's classical papers “Funktion und Begriff” and “Begriff und Gegenstand”. Frege's way of introducing concepts by first talking about function, may, however, only be a didactical device, as Frege can assume that his readers know functions in general and now have to realize the crucial role of concepts in logic.

the improper generalization that no saturated terms can have unsaturated entities as their referents. This simple syntax/semantics-isomorphism is unwarranted; an unsaturated entity of one ontic level might be considered as saturated enough to serve as argument for another unsaturated entity of the next ontic level (like in a hierarchy of functions). Frege's theory, however, can be amended to this purpose. We may add two further clauses to the theory above:

- ii. Singular terms of the form “the concept ___” refer to concepts.
- iii. Singular terms of the form “the concept ___” with the concept filling the slot being of order n saturate general terms referring to concepts of the next order, $n+1$.

We might also want to elucidate further the image of 'unsaturatedness' in its linguistic and ontological dimensions. [We address this elaboration of Frege's theory in the last two paragraphs.] Frege himself on the one hand made use throughout of functions/concepts being arguments of higher order functions/concepts (as the logical systems of the *Begriffsschrift* and the *Grundgesetze der Arithmetik* are variants of Second Order Logic, including also relational expressions relating objects to concepts). Frege, on the other hand, sternly denied that 'the concept *horse*' denotes a concept, landing himself in *Kerry's Paradox*. The latter denial of concepts as subject of a proposition or sentence was the main reason why Russell in *The Principles of Mathematics* did not follow Frege's theory of concepts. Russell substitutes 'propositional function' for Frege's 'concepts'. He denies Frege's categorical distinction between concept and objects and by this drops Frege's theory of sentential unity.

§4

Russellian propositional functions $\psi(\acute{e})$ and $\phi(\acute{e})$ are *equivalent* if propositions like $\psi(a)$ and $\phi(a)$ employing them have the same truth value for all arguments. $\psi(\acute{e})$ and $\phi(\acute{e})$ are *identical* if they have the same value for all their arguments, which means result in the same proposition respectively.³ The sentences expressing (1902) or being (1918) the proposition have, given the recursive truth conditions, the same truth value even though they do not designate truth values. As propositions

3 As propositional functions have propositions as values no intensionality is involved here.

can be identical a logic capturing all logical and metaphysical truths must include a sign of propositional identity and respective axioms which mirror the axioms of identity for objects. Because propositions can be arguments of propositional functions, Type Theory introduces type distinctions which may forbid a proposition being the argument of its 'own' propositional function, or make the propositional function 'systematically ambiguous' as it applies once to objects and once to propositions (i.e. entities of different types, as objects as not truth bearers are distinguished from propositions).

Whereas the equivalence of $\psi(\acute{e})$ and $\phi(\acute{e})$ can be as easily ascertained as the identity of sets by verifying the truth conditions of respective sentences, the question of identity for propositions taken as abstract objects is much harder to answer. How can we determine whether such propositions are identical? One criterion whether to base the metaphysics of logic on propositional functions could be the complexity of their identity conditions and our knowledge of them.

A simple solution would be at hand if the constituent structure of the sentences expressing these propositions corresponded to the constituent structure of the propositions. This meant that language – at least logical form – guides our metaphysical picture. And we are back at a Fregean theory of sentential unity.

§5

Russell's move from propositions as abstract entities *sui generis* to statements (assertive sentences) stems from problems of his early identity theory of truth and his refusal of sets/classes. In a proposition *sui generis* objects and relations/qualities are combined in some way. If they are combined in a straightforward sense of the relata really standing in the relation in question, there can only be true propositions as any proposition then corresponds – by identity – to a fact. A theory of the semantic content of false sentences thus is impossible.

Another way of combination could be the way several elements of a set are joined in their set membership. In this case the relation is one element and the relata are others, without the relation relating the other elements. A corresponding ontology models propositions not as entities *sui*

generis but as something like tuples (i.e. sets). As Russell wants to avoid sets – or at least admit them only as a manner of speaking about (predicative) propositional functions – he cannot model propositions this way. Therefore already in *Principia Mathematica* (1910) propositions as abstract entities *sui generis* give way to propositions as assertive sentences. Propositional functions have now to be understood as general terms/predicates.⁴

One should arrive then at the denotational theory that singular terms denote objects, general terms denote properties/qualities/concepts and assertoric sentences being evaluated as true or false only denote in case they are true. In this case they denote a fact. [One could otherwise claim that assertoric sentences do not denote at all, but in case of their truth correspond to a fact.] False sentences do not correspond to anything. The ensuing correspondence theory of truth then requires introducing facts either corresponding to true assertoric sentences of different logical form (e.g. disjunctive, universal etc.), or making assertoric sentences of non-atomic logical form in some truth conditional recursive fashion true.⁵

Truth bearers are, properly speaking, eternal assertoric sentences, either asserted themselves or going back to an asserted indexical declarative sentence. Indirectly one may consider the indexical sentences themselves as bearers of truth. A true assertoric sentence denotes a fact (corresponding to the referential content of the thought expressed by the sentence).

Properties ('universals' in some sense of that term) are abstract entities in the sense of being a structure or structural component of an object (a chunk of reality), i.e. abstract in as much as they have no independent existence. They are thus just the opposite of Platonic 'forms' ('abstract entities' in another sense). Them being abstract in this way doesn't make them non-existent, constructed by us and the like. We refer to them as structures of reality by our concepts. A general term is unsaturated *because* its referent is an abstract entity dependent on an object. The singular term referring to this object taken as argument of the general term in question yields a true atomic

4 This limits the cardinality of propositions and propositional functions to \aleph_0 – a problem for a foundation of mathematics, which we leave to the side here. Russell, being unaware of arithmetization, could not see this problem.

5 Russell's fact ontology in *The Philosophy of Logical Atomism* advances at least the fundamental building blocks of such an account. Russell, however, also set forth at that time the *Tractarian* doctrine that sentences are not part of the world – with all its bizarre consequences.

sentence.⁶

Such a theory would combine Frege's theory of concepts and objects with Russell's ontology of facts instead of having assertoric sentences denoting truth values.

We may picture facts by sentences, but facts do not become (true) sentences by this. Chunks of reality can be referred to by singular terms (and thus be considered as objects) or by true sentences (and thus be considered as facts). In this sense 'fact' and 'object' are ontological categories (i.e. categories employed within our ontological framework) covering the same pieces of reality. That doesn't mean in any sense that there either aren't objects or aren't facts. A chunk of reality has structure, taken the structure into consideration we have a fact, otherwise an object. That we 'take into consideration' again doesn't mean that it otherwise isn't there. Object talk refers to objects. Fact talk refers to facts. Some object talk and some fact talk can refer to the same chunk of reality. It is wrong to state that objects do not exist in reality, because we do not take into consideration (abstract away from) some structures present in them. Thus our ontology contains sentences (linguistic entities) and facts (chunks of reality) and no propositions *sui generis* in between or somewhere else.

§6

There are more difficulties with propositions as abstract entities.

If propositions are abstract entities, they are – at least once by the Theory of Descriptions reference to individuals has been eliminated – supposedly omnitemporal entities. They are, and they have always been there. How can we then reach out and contact them?

A theory of sentential unity may do some work here. Applying a general term to a singular term we also, in terms of meaning, relate the concept (or the relation) to the object (or the objects) in question – and *thus* grasp a proposition! Our account of our ability to create and understand sentences compositionally serves also as an account of our grasping of propositions. Our grasping is in this way also fine-grained enough to relate differently to propositions which, although logically

6 Russell's problems with sentential unity are aggravated by the absence of an account of universals/abstract entities. Acquaintance with universals need not mean by itself having the recognitional capacities to relate them to objects!

equivalent, carve up reality in different ways.

Contrary to this the idea of everlasting propositions invites a theory of grasping in which the grasping of a proposition is a single undivided act, since one needs a theory of such acts of grasping anyway for universals which are part of such an ontology of compound abstract propositions.

Sentential compositionality then becomes as *additional* issue, unrelated to grasping propositions.

A way out of this last dilemma consists in putting sentential unity and compositionality first in identifying propositions with sentences. Again a Fregean account carries the day.

In a Fregean theory of sentential unity applying a general term to a singular term corresponds as a linguistic activity to the semantic activity of relating the referent of the general term to the referent of the singular term. If one takes the referents of general terms to be abstract entities (in the sense above) this semantic activity consists in recognizing or surmising the abstract entity as having the object as carrier or bearer (in case of general terms with one argument) or as recognizing or surmising the abstract entity as supervening on properties of the relata (in case of general terms with more than one argument). The ensuing unity of a sentence corresponds in case the sentence is true to the unity of the fact. Substituting an ontology of facts for Frege's ontology of truth values – and the accompanying peculiar thesis of statements denoting truth values – one can extend Frege's theory of sentential unity and increase its explanatory power, tying it more closely to a viable ontology.

Analysis of language (of sentential unity) and ontological analysis (of facts) proceed in unison, with linguistic analysis leading the way. Frege thereby appears once more as the true founding father of analytic philosophy, Russell's attempted ontological foundation turned out to be a blind alley.